Tensor charge and anomalous magnetic moment correlation[1]

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I – Introduction.

Karl-Sehgal[i] formula (upgraded first by Cheng and Li[ii] then by Di Qing[iii] et al.) relating baryon magnetic moments to the spin structure of the constituent quarks takes into account the relativistic nature of quarks inside the parent nucleon. The upgraded formula by Di Qing et al. is a model independent, field theoretical relation which includes quark tensor charges in addition to the longitudinal spin part of the formula. The transverse spin structure is an independent structure at the relativistic level , with respect to the longitudinal spin structure[iv]. A straightforward but however lengthy way to obtain the formula is to expand quark field operators in nucleon matrix elements of quark currents in terms of a complete set of quark and antiquark wave functions. In performing such expansion, quark-antiquark pairs become operating if the baryon state is a Fock decomposition beyond the q^3 state. $|B\rangle = c_0\,|q^3\rangle + \sum_\alpha c_\alpha\,|q^3 q\bar{q}\rangle_\alpha + \cdots$ Attempts have been made to generalize the formula by taking into account the contributions from quark-antiquark pairs in a constituent quark model with valence q^3 and sea $q^3 q\bar{q}$ mixing. It is found that pair creations only contribute a small amount to the magnetic moment of the proton ($-0.065\ n.m$ with $n.m$ the nucleon magneton) . It is to note that the inclusion of sea quarks by authors of reference (iii) through the Fock space configuration is a tentative to include quark interactions into the scheme. In this paper we reconsider the problem of introducing interactions into the baryon magnetic moments formula by using a standard approach in which the baryon has the standard q^3 configuration .There are several possible sources of interactions which contribute to the baryon magnetic moments. Exchange magnetic moments[v] [vi] (they are generic in any interacting field theory), transition moments and individual anomalous magnetic moments (a.m.m) of quarks .Exchange magnetic moments contribute a non-additive piece to the baryon magnetic moments. This means that this contribution will add an additional term not proportional to the sum of individual quark magnetic moments. In the chiral quark model for instance, the two- body exchange moments (to consider only the leading)come from the exchange of one Nambu -Goldstone boson with one photon attached in all possible ways. A rough estimate of the size of exchange moments yields $0.010\ n.m$ [6].The exchange

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[1] For a Full analysis please refer to the recent paper of the author Phys.Rev.D 72, 114014(2005)

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correction, being connected with exchange of charged pions, requires the presence of u and d quarks in the baryon and hence contributes only to proton and neutron. Transition moments add a yet small piece to the process $\Sigma \rightarrow \Lambda$ .Other interactions are due to anomalous magnetic moments of quarks which on the contrary may contribute a significant amount. Nonlinear chiral quark model for instance may be used to estimate the order of magnitude of the anomalous contribution. In fact one would expect an anomalous magnetic moment[vii] of order $\frac{m^2{}_i}{\Lambda^2{}_{CSB}} \approx 10\%$ $(m_i \approx 360 Mev,\ \Lambda_{CSB} \approx 1 Gev)$ with m_i being the constituent mass of the quark, supposed to be the effect of chiral symmetry breaking, and Λ_{CSB} is the chiral symmetry breaking scale. There are several theoretical and experimental studies indicating quarks do have non negligible a.m.m. To fit the measured magnetic moment of the baryon octet, it is found that quarks must have a sizable a.m.m. In effect, non relativistic constituent quark model for light hadrons, with the measured anomalous magnetic moments for the proton and the neutron respectively $a_p = 1.79$ and $a_n = -1.91$ yields the relations.

$$\frac{1}{3}(4\mu_u - \mu_d) = 2.79$$
$$\frac{1}{3}(4\mu_d - \mu_u) = -1.91$$

From which we infer the measured quantities $\frac{M}{1+a}$.

$$\frac{m_u}{1+a_u} = 338\ Mev$$
$$\frac{m_d}{1+a_d} = 322\ Mev \qquad (1)$$

On the other hand, applying the constituent quark model to fit the hadron spectrum, required masses of the order $m_u \simeq m_d = 420\ Mev$ .Such values of masses suggest a sizable anomalous magnetic moments of the order $a_u \simeq 0.24,\ a_d \simeq 0.30$ and a small difference $a_u - a_d \simeq 0.07$ to recover the isospin symmetry. Bicudo et al[viii] have shown in several effective quark models that in the case of massless-current quarks, chiral symmetry breaking usually triggers the generation of an anomalous magnetic for the quarks of the order $a \simeq 0.28$ .In the same spirit, Singh [ix] has also proven that, in theories in which chiral symmetry breaks dynamically, quarks can have a large a.m.m. On the other hand, Köpp et al [x] have provided a stringent bound on the a.m.m from high-precision measurements at LEP, SLC, and HERA. In the second section we will give a theoretical argument showing that a.m.m are correlated to tensor charges and should necessarily accompany them.

In the following we assume we have derived an effective lagrangian defined at the scale of low-energy magnetic moments after having integrated all unwanted fields . Constituent quarks have masses m_i $i = u, d, s$ and do have anomalous magnetic moments from the term

$\frac{a_i Q_i}{2m_i}\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}$ in the effective lagrangian. Baryon magnetic moments $\vec{\mu}_N$, due solely to quark electric charges and their longitudinal spins, neglecting quark-antiquark pairs are by definition

$$\vec{\mu}_N = \langle PS \,|\, \sum_{i,\bar{i}} \frac{Q_i}{2} \int dr^3 \vec{r} \times \bar{\psi}_i \vec{\gamma} \psi_i \,|\, PS \rangle \qquad (2)$$

Q_i , $i = u, d, s$ are quark charges, $\psi_i\,(\psi_{\bar{i}})$ constituent quark (antiquark) fields and $|PS\rangle$ is the baryon ground state with momentum P and spin polarization S .The spin structure of quarks is encoded in the axial and tensor charges, respectively denoted $\Delta i = \Delta_i + \Delta_{\bar{i}}$ and $\delta i = \delta_i - \delta_{\bar{i}}$ (the minus sign accounts for the odd charge conjugation parity of the transverse spin operator) $\Delta_i\,(\Delta_{\bar{i}})$ are related to the expectation value of the relativistic quark(antiquark) spin operator in the baryon

$$\langle PS \,|\, \int dx^3 \psi_i^{\dagger} \vec{\Sigma} \psi_i \,|\, PS \rangle = 2\Delta_i \vec{S} \qquad (3)$$

Δ_i can also be shown to be related in the parton infinite momentum frame to the integrated helicity difference $\Delta_i = \int dx \left[q_{i\uparrow}(x) - q_{i\downarrow}(x) \right]$ with $q_{i\uparrow}(x), (q_{i\downarrow}(x))$, the probability of finding a quark with fraction x of the baryon momentum and polarization parallel (anti parallel) to the baryon spin .Similarly δ_i is given by the formula

$$\langle PS \,|\, \int dx^3 \bar{\psi}_i \vec{\Sigma} \psi_i \,|\, PS \rangle = \vec{\delta}_i \qquad (4)$$

The tensor charge is shown to be related to the first moment of the integrated quark transversity distribution $\delta_i = \int_0^1 dx \,[q_{\rightarrow i}(x) - q_{\leftarrow i}(x)]$ [xi] .Similar expressions apply to the antiquark. Unpublicized quark distribution (well known), quark helicity distribution(known), and transversity distribution (unmeasured but calculated on lattice, and several other models), provide together, a complete description of the quark spin .To stress the difference between helicity and transversity , recall that if quarks moved non relativistically in the nucleon, $\delta_i(x)$ and $\Delta_i(x)$ would be identical as only large components of the fermion field are leading in which case $\bar{\psi} = \psi^\dagger \gamma^0 \approx \psi^\dagger$ and both definitions (3) and (4) coincide . Another way of seeing this, is that rotations and Euclidean boosts commute and a series of boosts and rotations can convert a longitudinally polarized nucleon into a transversely polarized nucleon at infinite momentum. So the difference between transversity and helicity distributions reflects the relativistic motion of quarks inside the nucleon.

To express the baryon magnetic moment in terms of the spin degrees of freedom we compute (2) using the field current $\vec{j}_i = \bar{\psi}_i \vec{\gamma} \psi_i$ and assume the ground state of the baryon to have a vanishing non-relativistic orbital magnetic moment. To this end it is useful to decompose the quark current into two distinct pieces using Gordon decomposition and not to expand quark field operators in nucleon matrix elements of quark currents in terms of a complete set of quark and antiquark wave functions as in previous cited work . The convection current part and the spin current part contribute differently, giving respectively

$$\frac{xQ_i}{4m_i(1+x_i)}(\Delta_i - \frac{\delta_i}{x_i}) \qquad \frac{xQ_i}{4m_i}(\Delta_i + \frac{\delta_i}{x_i}) \tag{5}$$

where $x_i = \frac{m_i}{\langle E_i \rangle}$ is the ratio of the constituent quark mass to the average kinetic energy of the quark in the baryon ground state. Adding antiquarks and denoting $\vec{\mu}_N = \langle P \uparrow | \vec{\mu}_N | P \uparrow \rangle$ we get.

$$\begin{aligned} \mu_N &= \sum_{i=u,d,s} \mu_i W_i + ..., \\ \mu_i &= \frac{Q_i}{2m_i} \\ 2\frac{W_i}{x_i} &= \frac{1}{(1+x_i)}(\Delta_i - \Delta_{\bar{i}} - \frac{\delta i}{x_i}) + (\Delta_i - \Delta_{\bar{i}} + \frac{\delta i}{x_i}) \end{aligned} \tag{6}$$

Where dots represent possible collective contributions such as exchange moment and transition moment contributions we introduced earlier . Equation (6) is the weighted sum of two distinct combinations $(\Delta_i - \Delta_{\bar{i}} - \frac{\delta i}{x_i})$ and $(\Delta_i - \Delta_{\bar{i}} + \frac{\delta i}{x_i})$ .The former combination shrinks to zero in the non relativistic limit. The latter combination survives the non relativistic limit and has the advantage that it is the only one which will be affected by the anomalous magnetic moments of the quarks. Equation (6) is the upgraded Karl-Sehgal formula cited in reference iii but obtained in another rearrangement of terms. Their formula serves as a check to our Gordon decomposition in terms of the convection and the spin current.In Gordon decomposing the magnetic moment, the spin part takes the form $\vec{\mu}\big|_{spin} \propto \int \frac{1}{2m}\partial_\nu(\overline{\psi}\vec{\sigma}^\nu\psi)$ where $\vec{\sigma}^\nu$ is a vector which components are $\sigma^{i\nu}$ .The spatial derivative ∂_i gives (after neglecting a total derivative) the term $\int \frac{1}{2m}\overline{\psi}\vec{\Sigma}\psi$ while the time derivative ∂_0 gives a non vanishing contribution, as quark fields do depend on time . It is to be noted at this point that the authors of reference [xii] do not consider the time derivative[3] contribution which gives a

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[3] Cited authors retain only the $\vec{\Sigma}$ term

vanishing contribution if taken between true states of the nucleon but these true states are unreachable due to the strong dynamics of quarks and gluons[4] .

II – Tensor charge and anomalous magnetic moment correlation.

Let us have a close look to formula(6). This formula has an insufficiency .It leads to an absence of magnetism in the ultra-relativistic limit due in part to the fact that, it is the average energy of the quark inside the baryon that builds up the intrinsic magnetic moment and not the constituent mass m_i i.e. $x\mu_i \sim \frac{m_i}{\langle E_{i0}\rangle}\frac{1}{m_i} = \frac{1}{\langle E_{i0}\rangle}$ which goes to zero for infinite kinetic energy. The reduction factor x is explicit in(6) and is simply the Lorentz-Fitzgerald contraction length due to the relativistic boost as the magnetic moment is a vector (space components of a four vector). On the other hand, tensor charges in the formula, being there to account for constituent quark masses (the mass term $m\bar{\psi}\psi$ flips helicity and hence involves transversity), should also disappear in this limit. We have indeed $\mu_N\,|_{ultra} \sim (-\delta i + \delta i) = 0$ .The absence of magnetism in this limit suggests that formula(6) does have a missing term and that this term is associated with the anomalous magnetic moment of the quark. Why did we say that the anomalous magnetic moment of the quark is the missing term?. Formula(6) is a relativistic formula which describes how a magnetic photon couples to quarks being spinning point like objects. It also says that this coupling is decreasing with energy due to the reduction factor . On the other hand we know from quantum mechanics that particles of definite energy and momentum are not localized. It then follows a possible current in the lagrangian of the form[5]

$$\frac{\partial_\alpha \bar{\psi}\sigma^{\alpha\beta}\psi_\beta}{m} \tag{7}$$

Perturbatively, for a photon to probe such a current, a quark should radiate a field (gluon or goldstone boson or whatever) at position x and reabsorbed at a distant position y , once it interacts with the photon (vertex interaction and not a self-energy interaction).In this process the probing photon sees the quark as an extended object or rather an electric current circulating in the area of the extension .This is what we call “anomalous” magnetism. The correlation of the anomalous magnetic moment to the tensor charge is suggested by the structure of the current (7) which, as the mass term, flips helicity.

Adding quark anomalous magnetic moments of quarks to formula(6), this one generalizes to .

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[4] The authors of reference iii reacted to my publication Phys.Rev.D 72, 114014(2005) in which I raised the question in relation to the time component of the quark field by publishing a comment in Physical Review D soon to appear

[5] Differentiation of the field is non zero only if the field has a spatial and/or temporal extension. Point like objects have a current without derivatives such as $\bar{\psi}\gamma_\alpha\psi$ for instance.

$$\mu_N = \sum_{i=u,d,s} \mu_i W_i + ...,$$
$$2\frac{W_i}{x_i} = \frac{1}{(1+x_i)}(\Delta_i - \Delta_{\bar{i}} - \frac{\delta i}{x_i}) + (1+a_i)(\Delta_i - \Delta_{\bar{i}} + \frac{\delta i}{x_i}) \tag{8}$$

There is another different way of seeing that quark anomalous moments are missing . Let us rearrange formula(6) as this.

$$2W_i = A_i(\Delta_i - \Delta_{\bar{i}}) + B_i(\delta_i - \delta_{\bar{i}}) \tag{9}$$

Parameters A_i and B_i are expressed in terms of x_i .

$$\frac{A_i}{x_i} = \frac{1}{1+x_i} + 1$$
$$B_i = -\frac{1}{1+x_i} + 1 \tag{10}$$

Being functions of only one common parameter x_i , A_i and B_i are not independent parameters .Hence, these parameters could not distinguish between the contribution to baryon magnetic moments coming from helicities and the contribution coming from transversities, while these are supposed to be independent contributions in a relativistic regime. In general one may imagine that having two different spin structures in relativistic physics, namely, the longitudinal spin $\Delta_i, \Delta_{\bar{i}}$ and the transverse spin $\delta_i, \delta_{\bar{i}}$, quarks necessarily would carry two different magnetisms respectively of the form $\mu_i A_i(\Delta_i - \Delta_{\bar{i}})$ and $\mu_i B_i(\delta_i - \delta_{\bar{i}})$ [6].So in the relativistic case ,the most general contribution to the baryon magnetic moments of quarks and antiquarks would be of the form(9) but where A_i and B_i are two independent parameters .Identifying coefficients of axial and tensoriel magnetic densities in both (8) and (9)we get two independent parameters .

$$\frac{A_i}{x_i} = \frac{1}{1+x_i} + 1 + a_i$$
$$B_i = -\frac{1}{1+x_i} + 1 + a_i \tag{11}$$

We understand that the introduction of anomalous magnetic moment is a necessary requirement of relativity, otherwise parameters A_i and B_i would be dependant parameters (i.e. depend only on one parameter x_i) which means that helicity and transversity would no longer be two different spin structures in relativity. On the other it becomes also clear in this

[6] Hereafter we will call the first, axial magnetism (although it is not the axial charge $\Delta_i + \Delta_{\bar{i}}$ (sum) which is involved but $\Delta_i - \Delta_{\bar{i}}$ (difference)) and the second, tensoriel magnetism.

approach , that the quark anomalous magnetic moment is correlated to the quark transversity. Such a correlation is manifest at the ultra relativistic at which W_i function in (8) takes the form.

$$2W_i = a_i\,(\delta i)_{ultra} \tag{12}$$

where $(\delta i)_{ultra} = \frac{2}{3}\delta_i^{NR} = \frac{2}{3}\Delta_i^{NR}$ is the ultra relativistic limit .This limit makes it explicit that quark anomalous magnetic moments together with tensor charges dominate the ultra relativistic regime.

III – Baryon magnetic Moments Analysis and Numerical Applications

Refer to the recent paper of the author Phys.Rev.D 72, 114014(2005) for this section.

IV- Conclusion

Magnetic moments of the nucleon are static properties (nucleon at rest). The quark inside the nucleon are nevertheless strongly bound relativistic objects. Being relativistic, the spin structure of quarks involves in general, both quark helicity distributions and quark transversity distributions. Latter distributions encode relativistic effects of quarks inside the nucleon. We have shown in this study that since relativity requires existence of two independent spin structure, one longitudinal and the other transverse, it then follows, the existence of two independent magnetisms which we may call respectively axial and tensoriel. The contribution of each component is weighted by two independent parameters namely $0 \prec x_i \prec 1$ the ratio of the quark constituent mass to the quark average kinetic energy, and the anomalous magnetic moment a_i .Hence the quark anomalous magnetic moment a_i is strongly correlated to the tensor charge δi and this correlation is made more explicit in the ultra relativistic limit. Sehgal-Karl-Chen formula relating baryon magnetic moments to the quark spin is a relativistic formula which necessarily includes quark tensor charges, but according to the above considerations such formula is lacking an essential ingredient which is the quark anomalous magnetic moments which are correlated to tensor charges. To get a consistent formula for baryon magnetic moments we do add the missing part. We then confronted our formula with baryon magnetic moments data using reasonable inputs such as $\frac{\mu_u}{\mu_d} = -2$, $m_u \simeq 263\ Mev$, considerations from Melosh –Wigner rotation reductions of nucleon spin to estimate x_u, x_d and tensor charges from various model computations .The outcome is large enough anomalous magnetic moments, difficult to ignore.

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