Transverse-Momentum-Dependent Functions in Semi-Inclusive DIS

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Essential literature

- *Mulders, Tangerman, NPB 461 (96)*
- *Boer, Mulders, PRD 57 (98)*
- *Bacchetta, Mulders, Pijlman, PLB 595 (04)*
- *Goeke, Metz, Schlegel, PLB 618 (05)*

Need to review and complete the study of one-particle inclusive DIS at low transverse momentum (soon on the arXiv…)

Outline

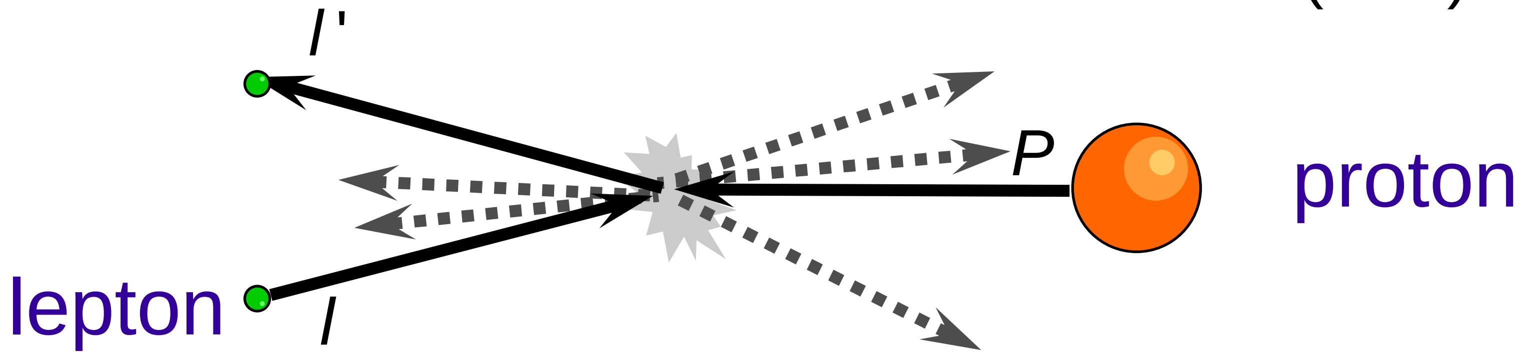
- Inclusive and semi-inclusive DIS
- Cross section in terms of structure functions
- Selected examples of structure functions in terms of transverse-momentum-dependent (TMD) partonic functions
- Conclusions

Inclusive DIS

$$\ell(l) + p^{\uparrow}(P) \to \ell(l') + X$$

$-(l - l')^2 = Q^2 =$ virtuality of photon

$$x = \frac{Q^2}{2P \cdot (l - l')}$$

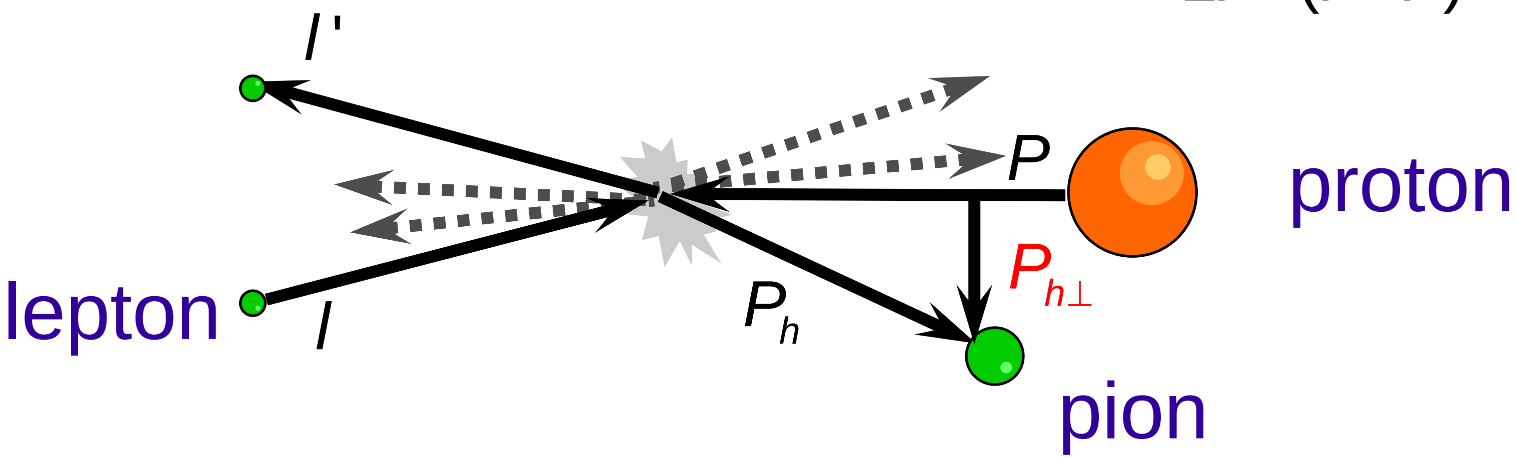


Semi-inclusive DIS (SIDIS)

$$\ell(l) + p^{\uparrow}(P) \to \ell(l') + h(P_h) + X$$

$-(l - l')^2 = Q^2 =$ virtuality of photon

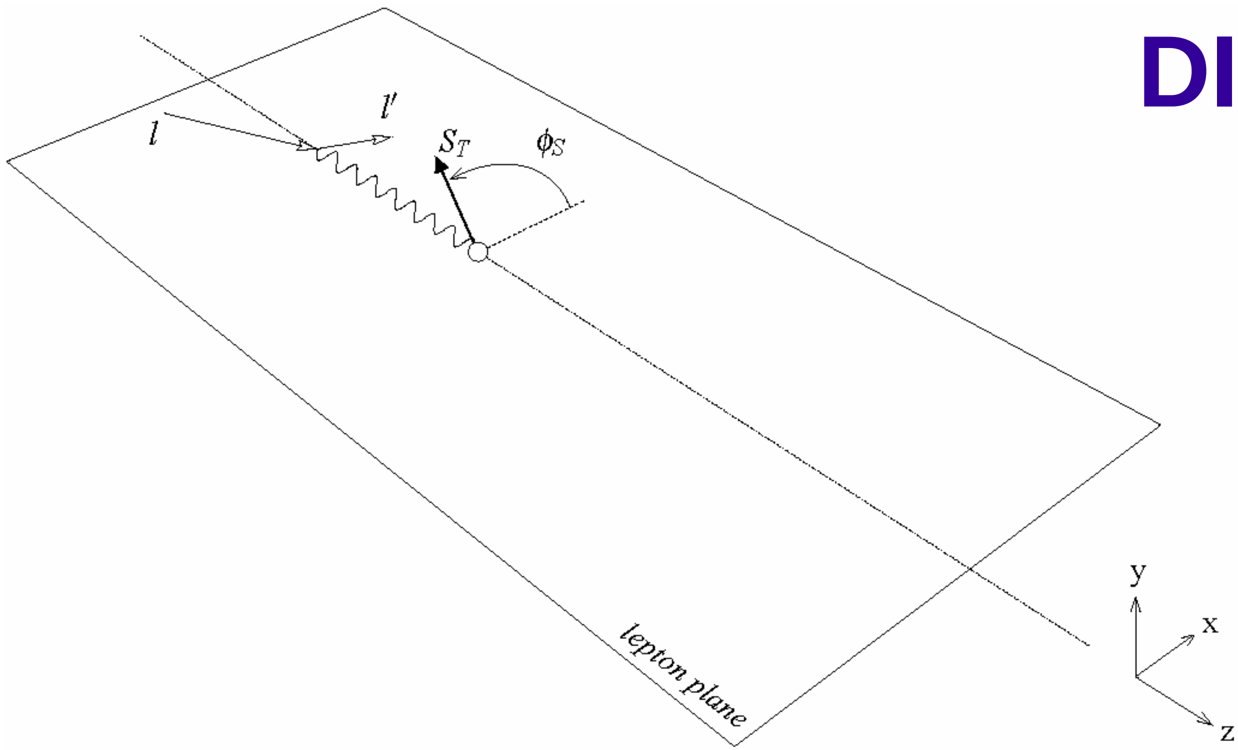
$$x = \frac{Q^2}{2P \cdot (l - l')}$$



$$z = \frac{P \cdot P_h}{P \cdot (l - l')}$$

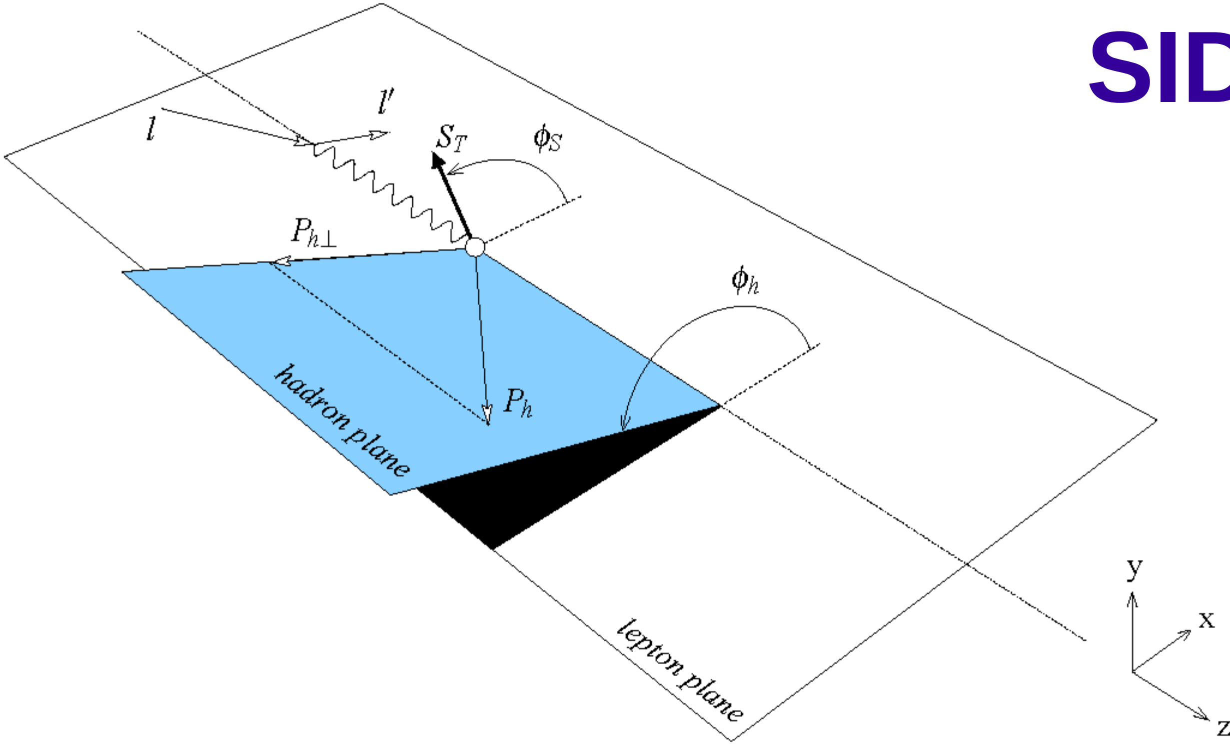
$P_{h\perp} =$ transverse momentum of pion

Vectors and angles involved



DIS

Vectors and angles involved



SIDIS

Cross-sections

DIS

$F_T(x,Q^2)$

$$\frac{d\sigma}{dx\,dy\,d\phi_S} = \frac{2\,\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\,\Big\{F_T + \varepsilon\,F_L + S_L\,\lambda_e\,\sqrt{1-\varepsilon^2}\;2x\,(g_1 - \gamma^2\,g_2) + S_T\,\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,2\,x\gamma(g_1+g_2)\Big\}$$

SIDIS integrated over $P_{h\perp}$

$F_{UU,T}(x,z,Q^2)$

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz} = \frac{2\,\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\,\Big\{F_{UU,T} + \varepsilon\,F_{UU,L} + S_L\,\lambda_e\,\sqrt{1-\varepsilon^2}\;F_{LL} + S_T\,\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + S_T\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}\Big\}$$

Full SIDIS cross section

Kotzinian, NPB 441 (95)
Diehl, Sapeta, EPJC 41 (05)

$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Big\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$
$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\Big]$$
$$+ S_L\,\lambda_e\Big[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\Big] + S_T\Big[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big)$$
$$+ \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$
$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] + S_T\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}$$

18 structure functions   *9 already measured!*

Regime $P_{h\perp} \ll Q$

- Structure functions can be interpreted as convolutions of TMD partonic functions

- Factorization proof (leading twist)

Ji, Ma, Yuan, PRD 71 (04)

- Subleading twist is included, even though factorization is not guaranteed

- For large $P_{h\perp}$ see talks by *Vogelsang, Yuan, Koike, Tanaka*

Unpolarized beam and target

$$F_{UU,T} = \mathcal{C}\left[f_1 D_1\right]$$

$$\mathcal{C}[f_1 D_1] = x\sum_q e_q^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z) f_1^q(x,p_T^2) D_1^q(z,k_T^2)$$

$$F_{UU,L} = 0$$

Cahn effect

$$F_{UU}^{\cos\phi_h} = \frac{M}{Q}\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xh\,H_1^{\perp} + \frac{M_h}{M} f_1 \frac{\tilde{D}^{\perp}}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xf^{\perp} D_1 + \frac{M_h}{M} h_1^{\perp}\frac{\tilde{H}}{z}\right)\right]$$

ZEUS, hep-ex/0608053

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\,\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h} h_1^{\perp} H_1^{\perp}\right]$$

ZEUS, hep-ex/0608053

Boer-Mulders

Collins

Longitudinally pol. beam/target

$$F_{LU}^{\sin\phi_h} = \frac{M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xe\,H_1^\perp + \frac{M_h}{M}\,f_1\,\frac{\tilde{G}^\perp}{z}\right) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg^\perp D_1 + \frac{M_h}{M}\,h_1^\perp\,\frac{\tilde{E}}{z}\right)\right]$$

CLAS, PRD 69 (04)

$$F_{UL}^{\sin\phi_h} = \frac{M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xh_L\,H_1^\perp + \frac{M_h}{M}\,g_{1L}\,\frac{\tilde{G}^\perp}{z}\right) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xf_L^\perp D_1 - \frac{M_h}{M}\,h_{1L}^\perp\,\frac{\tilde{H}}{z}\right)\right]$$

HERMES, PLB 622 (05)

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\,\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}\,h_{1L}^\perp H_1^\perp\right]$$

HERMES, PRL 84 (00), PLB 562 (03)

$$F_{LL} = \mathcal{C}\left[g_{1L} D_1\right]$$

HERMES, PRD 71 (05)

$$F_{LL}^{\cos\phi_h} = \frac{M}{Q}\,\mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xe_L H_1^\perp - \frac{M_h}{M}\,g_{1L}\,\frac{\tilde{D}^\perp}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg_L^\perp D_1 + \frac{M_h}{M}\,h_{1L}^\perp\,\frac{\tilde{E}}{z}\right)\right]$$

See talk by Kotzinian

Transversely pol. target (selection)

Sivers

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\boldsymbol{p}_T\cdot\hat{\boldsymbol{h}}}{M} f_{1T}^{\perp} D_1\right]$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\boldsymbol{k}_T\cdot\hat{\boldsymbol{h}}}{M_h} h_1 H_1^{\perp}\right]$$

Transversity

HERMES, PRL 94 (05)

COMPASS, PRL 94 (05)

See talks by Diefenthaler, Bradamante, Yuan, Efremov, Prokudin

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[\frac{\boldsymbol{p}_T\cdot\hat{\boldsymbol{h}}}{M} g_{1T} D_1\right]$$

See talk by Kotzinian

$$F_{UT}^{\sin\phi_S} = \frac{M}{Q}\mathcal{C}\left\{\left(x f_T D_1 - \frac{M_h}{M} h_1 \frac{\tilde{H}}{z}\right) - \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x h_T H_1^{\perp} + \frac{M_h}{M} g_{1T}\frac{\tilde{G}^{\perp}}{z}\right) - \left(x h_T^{\perp} H_1^{\perp} - \frac{M_h}{M} f_{1T}^{\perp}\frac{\tilde{D}^{\perp}}{z}\right)\right]\right\}$$

Transversely pol. target (selection)

$$F_{UT}^{\sin\phi_S} = \frac{M}{Q}\,\mathcal{C}\Big\{\Big(xf_T D_1 - \frac{M_h}{M}\,h_1\,\frac{\tilde{H}}{z}\Big) - \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(xh_T H_1^{\perp} + \frac{M_h}{M}\,g_{1T}\,\frac{\tilde{G}^{\perp}}{z}\Big) - \Big(xh_T^{\perp} H_1^{\perp} - \frac{M_h}{M}\,f_{1T}^{\perp}\,\frac{\tilde{D}^{\perp}}{z}\Big)\Big]\Big\}$$

Integrated over $P_{h\perp}$

$$F_{UT}^{\sin\phi_S} = -x\sum_q e_q^2\,\frac{M_h}{Q}\,h_1^q(x)\,\frac{\tilde{H}^q(z)}{z}$$

In inclusive DIS

$$F_{UT}^{\sin\phi_S} = 0$$

Conclusions

- In one-particle-inclusive DIS there are 18 structure functions to be measured
- Some information about 9 of them is already available
- The structure functions can be interpreted in the low-transverse-momentum regime in terms of transverse-momentum-dependent partonic functions
- Progress in experiments, phenomenology and theory is flourishing