Polarisation Observables in Proton Antiproton to Lepton Antilepton Reactions

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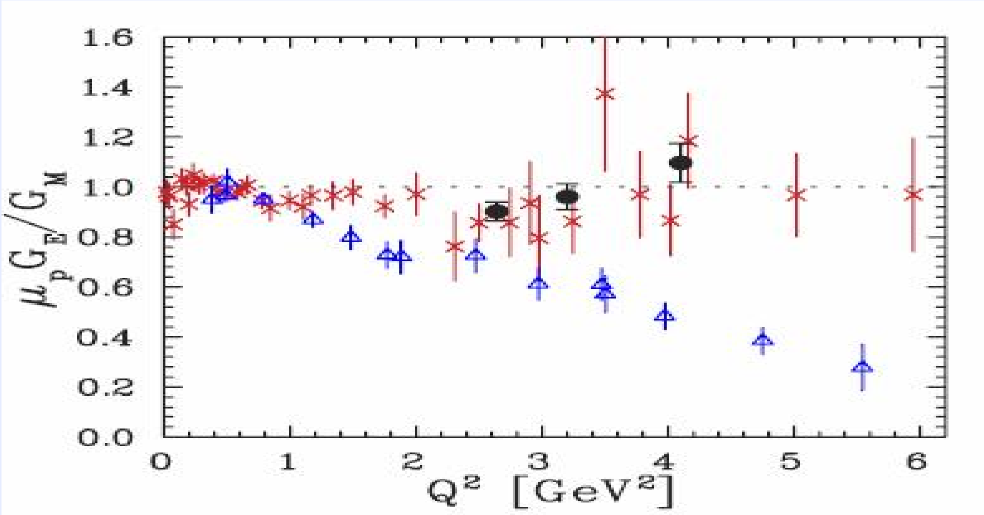
Introduction

- Electromagnetic form factors in the time like region are largely unknown.
- Relative phases have never been measured in the proton case.
- Proton form factor G_M measured up to 31 GeV^2 in space like region.
- In time like region from $\bar{p}\,p$ or $e^+\,e^-$ annihilation up to $q^2 = 5.6$ GeV^2.
- They show G_M follows approximately a dipole form:

$$G_E(q^2) = \frac{G_M(q^2)}{\mu_p} = G_D = \frac{1}{\left[1 + q^2/0.71\,(\mathrm{GeV}^2)\right]^2}$$

Proton G_E/G_M ratio



- CERN measurements (red) consistent with scaling $G_M(Q^2) \cong \mu_p G_E(Q^2)$

- Jlab measured the space like form factors using polarisation transfer. The ratio $G_E(Q^2)/G_M(Q^2)$ decreases at large Q^2 (blue).

- Coefficient $-q^2/4\,M^2$ of ${G_M}^2$ inhibits contribution of ${G_E}^2$ to $d\sigma/d\Omega$

- Uncertain radiative corrections - Rosenbluth method unreliable?

Form Factors in the Time Like Region

- Fermilab E835: G_M in time like twice as large as in space like region.
- Space like form factors of a stable hadron are real, on the real axis.
- Time like form factors have a phase above threshold, in general.
- Enhancement of G_M due to final state interaction of outgoing hadrons?
- QCD predicts asymptotic behaviour for G_M of the form

$$G_M\left(Q^2\right) = \frac{c}{s^2}\left(\ln\frac{s}{\Lambda^2}\right)^{-2}$$

Antiprotons

- Fermilab - polarised ($\sim 45\%$) $\bar{p}$ beams produced in weak decay of $\bar{\Lambda}$
- Need greater luminosity of $\bar{p}$ to probe antiquark structure.
- Future antiproton facility, FAIR at GSI, Darmstadt.
- *PAX* Collaboration seeks moduli and relative phases of G_E and G_M.
- Measurement of phases may explain q^2 dependence of G_E/G_M.

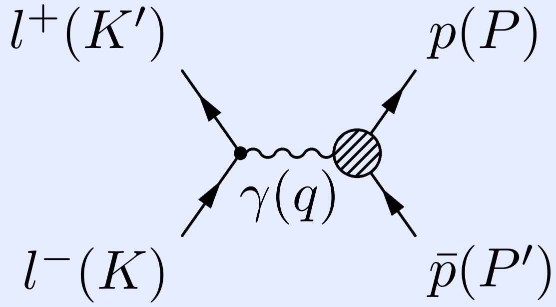
www.fz-juelich.de/ikp/pax/

Proton Form Factors

- G_E and G_M relate to Fourier transforms of nuclear charge and magnetisation density distributions.
- Dispersion relations connect the space (negative s) and time like regimes.
- Phases measured via single spin asymmetry in $\bar{p}\,p \to e^+\,e^-$.
- Understanding the onset of pQCD asymptotics
- Fundamental tests of dispersion theory and analyticity.
- Double spin asymmetry: limit relative phase ambiguity

  independent G_E–G_M separation

Internal Structure of the Proton



Lorentz and gauge invariance provide the general on-shell proton current

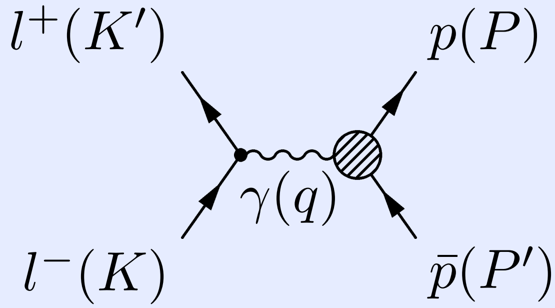
$$J_\mu \;=\; e\,\bar{u}(P)\left(F_1(s)\,\gamma_\mu + i\,F_2(s)\,\frac{\sigma_{\mu\nu}\,q^\nu}{2\,M}\right)v(P')$$

where $q_\nu = P_\nu + P'_\nu$ for s channel $\bar{p}\,p$ process. Dirac and Pauli form factors are normalised $F_1(0) = 1$ and $F_2(0) = \mu_p - 1$, the anomalous magnetic moment and Sachs EM form factors are (with $\tau = s/4\,M^2$)

$$G_E \;=\; F_1 + \tau\,F_2\,, \qquad G_M \;=\; F_1 + F_2\,.$$

At threshold $G_E(4M^2) = G_M(4M^2)$ where M is the mass of the proton.

Spin Averaged Cross Section



The unpolarised differential cross section for s channel annihilation of spin half particles in the centre of mass system is

$$\frac{d\sigma}{d\Omega} = \frac{\beta}{64\pi^2 s}\frac{1}{4}\sum_{\text{spin}}|\mathcal{M}|^2$$

where $\mathcal{M}$ is the invariant amplitude for the process and β is a flux factor. In an annihilation reaction of two spinor particles of mass m_i producing a pair of mass m_f the flux factor β is given by

$$\beta = \sqrt{(s - 4\,m_f^2)/(s - 4\,m_i^2)}$$

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Spin Averaged Cross Section

The spin averaged differential cross section for $l^+ + l^- \to p + \bar{p}$ scattering in terms of Mandelstam variables s and t is:

$$\frac{d\sigma}{d\Omega} = \alpha^2\,\beta\,\frac{1}{s^3\,(s-4\,M^2)}\left\{\frac{s^2}{2}\left(s-4\,M^2\right)\,|G_M|^2\right.$$
$$-4\,s\,m^2\,M^2\,\left(\,|G_M|^2-|G_E|^2\,\right)$$
$$\left.+\left[\left(t\,-\,m^2-M^2\right)^2+s\,t\right]\,\left(\,s\,|G_M|^2-4\,M^2|G_E|^2\right)\,\right\}$$

where m is the mass of the lepton and M is the mass of the proton. The flux factor $\beta = \sqrt{s-4\,M^2}/\sqrt{s-4\,m^2}$ and $t = (P-K)^2$.

This and the following results has been verified using Mathematica and all would be required for studies of $\bar{p}\,p \to \mu^+\mu^-$ or $\tau^+\,\tau^-$.

Spin Averaged Cross Section

We can simplify the previous expression by neglecting the mass of the lepton.

$$\frac{d\sigma}{d\Omega} \quad = \quad \frac{\alpha^2}{4}\frac{\beta}{s}\left\{(1+\cos^2\theta)\,|G_M|^2 + \frac{1}{\tau}\sin^2\theta\,|G_E|^2\right\}$$

- Absolute values of the form factors can be determined by a Rosenbluth separation technique.
- Space like: measurements made at a number of angles θ, fixed q^2
- Time like: required to change only one kinematic variable, $\cos\theta$

Asymmetries with lepton mass

It is convenient to define a scaled unpolarised cross section given below in terms of s, t.

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4}\frac{\beta}{s} D$$

where D is given by:

$$D = \frac{16\,M^2}{s^2\,(s-4\,M^2)}\left\{\,\left[\left(t - m^2 - M^2\right)^2 + s\,t\right]\left(\frac{s}{4\,M^2}\,|G_M|^2 - |G_E|^2\right)\right.$$
$$\left. + \frac{s^2}{8\,M^2}\,\left(s-4\,M^2\right)\,|G_M|^2 - s\,m^2\,\left(|G_M|^2 - |G_E|^2\right)\right\}.$$

Single Spin Asymmetry

When the antiproton in $l^+ l^- \longrightarrow p\,\bar{p}$ is polarised and if the initial leptons are unpolarised we obtain one non zero single spin asymmetry using the polarisation vector $S_N = (0,\, 0,\, 1,\, 0)$.

The asymmetry parameter A_N is defined as a measure of the left-right asymmetry by

$$A_N \quad = \quad \frac{(d\sigma/d\Omega)_\uparrow \; - \; (d\sigma/d\Omega)_\downarrow}{(d\sigma/d\Omega)_\uparrow \; + \; (d\sigma/d\Omega)_\downarrow}$$

We then obtain the general $(m \neq 0)$ single spin asymmetry for either a polarised proton or antiproton.

$$A_N \quad = \quad \left(1 - \frac{4\,m^2}{s}\right) \frac{2\,M\ \sin 2\theta}{\sqrt{s}\,D}\,\mathrm{Im}\,G_E^* G_M$$

Single Spin Asymmetry

This is an example of how T-odd observables can be non zero if final state interactions give interfering amplitudes.

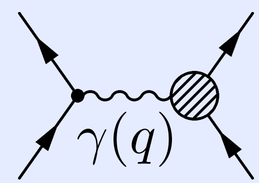
The other single spin observables A_S and A_L are non zero only when the initial lepton is polarised.

We can simplify A_N by neglecting the lepton mass to obtain the following expression.

$$A_N = \frac{\sin 2\theta \,\mathrm{Im}\, G_E^* G_M}{\left\{(1+\cos^2\theta)\,|G_M|^2 + \frac{1}{\tau}\sin^2\theta\,|G_E|^2\right\}\sqrt{\tau}}$$

This result is in agreement with A. Z. Dubnickova *et al.*, Nuovo Cim. A106 : 1253, (1993) and G. I. Gakh *et al.*, DAPNIA-05-293, Nov (2005).

Double Spin Asymmetry



The double spin observables involving normal (N), transverse (S) and longitudinal (L) spin directions are

$$
\begin{aligned}
A_{\mathrm{SS}} \quad = \quad & N \Big\{ \Big[\left(t - m^2 - M^2\right)^2 + s\,t \Big] \left(s\,|G_M|^2 - 4\,M^2 |G_E|^2 \right) \\
& + \frac{1}{2}\, s \,(\, s - 4\,M^2\,)\, \left(s - 4\,m^2 \right)\, \sin^2\theta\, |G_M|^2 \\
& - 4\, s\, m^2\, M^2\, \left(|G_M|^2 - |G_E|^2 \right) \Big\}
\end{aligned}
$$

Double Spin Asymmetry

$$A_{\rm NN} = N \Big\{ \Big[\big(t - m^2 - M^2 \big)^2 + s\,t \Big] \, \big(s\,|G_M|^2 - 4\,M^2\,|G_E|^2 \big)$$

$$-4\,s\,m^2\,M^2\,\big(|G_M|^2 - |G_E|^2 \big) \Big\}$$

$$A_{\rm LL} = N \Big\{ \Big[\big(t - m^2 - M^2 \big)^2 + s\,t \Big] \, \big(s\,|G_M|^2 + 4\,M^2\,|G_E|^2 \big)$$

$$+\frac{1}{2}\,s^2\,(\,s - 4\,M^2\,)\,|G_M|^2 + 4\,s\,m^2\,M^2\,\big(|G_M|^2 - |G_E|^2 \big) \Big\}$$

where the coefficient N is given by

$$N = \frac{4}{s^2\,(\,s - 4\,M^2\,)\,D}$$

Double Spin Asymmetry

The last two double spin observables, A_{SL} and A_{LN}, in terms of the centre of mass scattering angle are

$$A_{\mathrm{SL}} = \frac{2\,M}{\sqrt{s}\,D}\left(1 - \frac{4\,m^2}{s}\right)\sin 2\theta\,\mathsf{Re}\,G_E^* G_M$$

$$A_{\mathrm{LN}} = \frac{2\,M}{\sqrt{s}\,D}\left(1 - \frac{4\,m^2}{s}\right)\sin 2\theta\,\mathsf{Im}\,G_E^* G_M$$

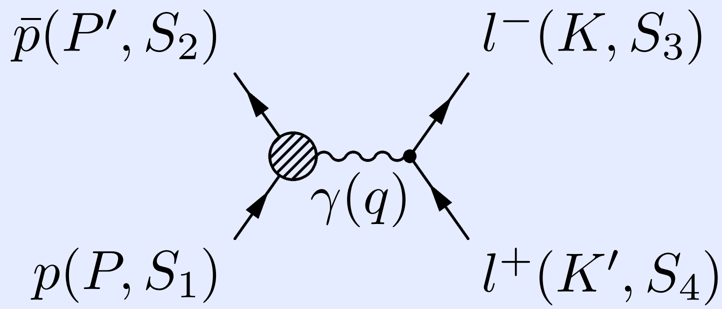
Polarisation observables can be used to pin down the relative phases of the time like form factors.

All of the double spin observables depend on the moduli squared of the form factors apart from A_{SL} and A_{LN} which contain the real and imaginary parts, respectively.

All of the above formulae reduce to previously published expressions when we let the lepton mass $m \to 0$. In the equations that follow $\tau = s/4\,M^2$.

$$
\begin{aligned}
A_{SN} &= 0 \\
A_{SL} &= \frac{\tau^{-1/2}\ \sin 2\theta\, \mathrm{Re}\, G_E G_M^*}{(1+\cos^2\theta)\,|G_M|^2 + \frac{1}{\tau}\ \sin^2\theta\,|G_E|^2} \\
A_{LN} &= \frac{-\tau^{-1/2}\ \sin 2\theta\, \mathrm{Im}\, G_E G_M^*}{(1+\cos^2\theta)\,|G_M|^2 + \frac{1}{\tau}\ \sin^2\theta\,|G_E|^2} \\
A_{SS} &= \frac{\sin^2\theta\ \left(\tau^{-1}\,|G_E|^2 + |G_M|^2\right)}{(1+\cos^2\theta)\,|G_M|^2 + \frac{1}{\tau}\ \sin^2\theta\,|G_E|^2} \\
A_{NN} &= \frac{\sin^2\theta\ \left(\tau^{-1}\,|G_E|^2 - |G_M|^2\right)}{(1+\cos^2\theta)\,|G_M|^2 + \frac{1}{\tau}\ \sin^2\theta\,|G_E|^2} \\
A_{LL} &= \frac{\left(1+\cos^2\theta\right)|G_M|^2 - \tau^{-1}\ \sin^2\theta\,|G_E|^2}{(1+\cos^2\theta)\,|G_M|^2 + \frac{1}{\tau}\ \sin^2\theta\,|G_E|^2}
\end{aligned}
$$

Helicity Amplitudes



We use the following spin four vectors:

$$S_1 = \frac{\epsilon_1}{M}(p,\, 0,\, 0,\, E) \qquad S_3 = \frac{\epsilon_3}{m}(k,\, E\sin\theta,\, 0,\, E\cos\theta)$$

$$S_2 = \frac{\epsilon_2}{M}(p,\, 0,\, 0,\, -E) \qquad S_4 = \frac{\epsilon_4}{m}(k,\, -E\sin\theta,\, 0,\, -E\cos\theta)$$

Polarisation dependent cross sections

P, T and C invariance indicate that there are five independent cross sections

$$\begin{aligned}
\frac{d\sigma}{d\Omega}(+\,+\,+\,+) &= \frac{d\sigma}{d\Omega}(+\,+\,-\,-) = \frac{d\sigma}{d\Omega}(-\,-\,-\,-) = \frac{d\sigma}{d\Omega}(-\,-\,+\,+) \\
\frac{d\sigma}{d\Omega}(+\,+\,+\,-) &= \frac{d\sigma}{d\Omega}(+\,+\,-\,+) = \frac{d\sigma}{d\Omega}(-\,-\,-\,+) = \frac{d\sigma}{d\Omega}(-\,-\,+\,-) \\
\frac{d\sigma}{d\Omega}(+\,-\,-\,-) &= \frac{d\sigma}{d\Omega}(+\,-\,+\,+) = \frac{d\sigma}{d\Omega}(-\,+\,-\,-) = \frac{d\sigma}{d\Omega}(-\,+\,+\,+) \\
\frac{d\sigma}{d\Omega}(+\,-\,+\,-) &= \frac{d\sigma}{d\Omega}(-\,+\,-\,+) \\
\frac{d\sigma}{d\Omega}(+\,-\,-\,+) &= \frac{d\sigma}{d\Omega}(-\,+\,+\,-)
\end{aligned}$$

Helicity Amplitudes

For $\bar{p}\,p \longrightarrow l^-\,l^+$ we have

$$\begin{aligned}
\frac{s}{\beta}\,\mathcal{M}\,(+\;+\;+\,+) &= 2\,\frac{m\,M}{s}\,\alpha\,\cos\theta\,|G_E| \\
\frac{s}{\beta}\,\mathcal{M}\,(+\;+\;+\,-) &= \frac{M}{\sqrt{s}}\,\alpha\,\sin\theta\,|G_E| \\
\frac{s}{\beta}\,\mathcal{M}\,(+\;-\;+\,-) &= \frac{\alpha}{2}\,(1+\cos\theta)\,|G_M| \\
\frac{s}{\beta}\,\mathcal{M}\,(+\;-\;-\,+) &= \frac{\alpha}{2}\,(1-\cos\theta)\,|G_M| \\
\frac{s}{\beta}\,\mathcal{M}\,(+\;-\;-\,-) &= \frac{m}{\sqrt{s}}\,\alpha\,\sin\theta\,|G_M|
\end{aligned}$$

Helicity Amplitudes

The spin averaged differential cross section given earlier can be written as

$$\frac{4\,s}{\alpha^2\,\beta}\,\frac{d\sigma}{d\Omega} \;=\; \left(1+\cos^2\theta\right)\,|G_M|^2 + \frac{4\,m^2}{s}\,\sin^2\theta\,|G_M|^2$$

$$+\frac{16\,m^2\,M^2}{s^2}\,\cos^2\theta\,|G_E|^2 + \frac{4\,M^2}{s}\,\sin^2\theta\,|G_E|^2$$

This is the following sum of the five independent helicity amplitudes listed above:

$$\frac{d\sigma}{d\Omega} \;=\; |\mathcal{M}\,(+\;+\;++)|^2 + |\mathcal{M}\,(+\;+\;+-)|^2 + |\mathcal{M}\,(+\;-\;--)|^2 +$$

$$\frac{1}{2}\,|\mathcal{M}\,(+\;-\;+-)|^2 + \frac{1}{2}\,|\mathcal{M}\,(+\;-\;-+)|^2$$

Summary

- For $\bar{p}\,p \to l^-\,l^+$ general expressions, including the lepton mass, for the spin averaged cross section, single and double spin asymmetries have been given.

- In spin averaged cross sections G_E and G_M contribute separately in the form of moduli in the time like region.

- The single spin asymmetry, A_N, is substantial and strongly discriminates between the analytic forms which fit the proton G_E/G_M data.

- Polarised antiprotons can explore the unexpected Q^2 dependence of the ratio of electric and magnetic form factors by studying their phases.

- In the case of mu and tau final state pairs it will be necessary to retain the lepton mass in the formulae for both the spin averaged differential cross section and the polarisation observables.

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