Asymmetry in Peripheral Spin Dependent Scattering

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Summary

- Spin frontier for the partonic structure of the nucleon
- Observables for polarized proton elastic collisions
- Spin dependence in hadronic and em elastic processes
- Peripheral longitudinal and transverse spin asymmetries
- Conclusions

Introduction

Polarized protons facilitate the study of spin dependent amplitudes for forward pp elastic scattering, particularly

$$\phi_+(s,0) \;\propto\; \sigma_{\rm tot}\,(i+\rho)\,, \qquad \phi_-(s,0) \;\propto\; \Delta\sigma_{_L}\,(i+\rho_-)$$

$$\phi_5(s,t) \;\propto\; r_5\sqrt{-t}\,, \qquad \phi_2(s,0) \;\propto\; \Delta\sigma_{_T}\,(i+\rho_2)$$

with the definitions, $\phi_\pm = (\phi_1 \pm \phi_3)/2$ where ϕ_+ refers to the dominant spin averaged proton proton amplitude.

References

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Scaled Amplitudes

Ratios of helicity amplitudes relative to the dominant imaginary elastic proton proton amplitude $\mathrm{Im}\,\phi_+$, with proton mass m,

$$R_2 + i\,I_2 = \frac{\phi_2}{2\,\mathrm{Im}\,\phi_+}$$

$$R_- + i\,I_- = \frac{\phi_-}{\mathrm{Im}\,\phi_+}$$

$$R_5 + i\,I_5 = \left(\frac{m}{\sqrt{-t}}\right)\frac{\phi_5}{\mathrm{Im}\,\phi_+}$$

are expected to have a less pronounced dependence on t.

To first order in small quantities ρ, R_5, I_5, and Coulomb phase δ, the maximum of the asymmetry A_N occurs near $\sqrt{3}t_c$, where

$$t_c = \frac{8\,\pi\,\alpha}{\sigma_{\text{tot}}}$$

and A_N has the following maximum value in t

$$\frac{4\,m\,A_{\max}}{\sqrt{-3\,\sqrt{3}\;t_c}} = \kappa\left[1 + \frac{\sqrt{3}}{2}\,(\rho + \delta)\right] - \,2\sqrt{3}\,R_5 - 2\,I_5\, + \,\cdots$$

with additional terms involving spin dependent hadronic quantities R_2, I_2, R_-, I_- that are accessible from asymmetry measurements undertaken in the interference region with both initial protons spin polarized.

Spin Asymmetries

A_N, A_{NN}, A_{SS}, A_{LL}, A_{SL} have singular terms in t arising from photon exchange. The scaled pp differential cross section

$$\mathcal{I}_0 \equiv \left(\frac{t}{e^{Bt}\,\sigma_{\rm tot}} \right) \frac{d\sigma}{dt}$$

with slope parameter B has an expansion in powers of t with constant coefficients a_0, and b_0.

$$\mathcal{I}_0 = \frac{4\pi}{\sigma_{\rm tot}} \frac{\alpha^2}{t} + \alpha\, a_0 + \frac{\sigma_{\rm tot}}{8\pi} b_0\, t + \cdots$$

For initial protons polarized along axis N normal to the scattering plane (and similarly for L longitudinal) note

$$A_{NN}\,\mathcal{I}_0 \;=\; \alpha\, a_{NN} + \frac{\sigma_{\rm tot}}{8\pi}\, b_{NN}\, t + \cdots$$

Spin observables with both initial protons polarized along perpendicular axes S, L have a similar power series expansion

$$-A_{SL}\,\mathcal{I}_1 \;=\; \alpha\, a_{SL} + \frac{\sigma_{\rm tot}}{8\pi}\, b_{SL}\, t + \cdots$$

In identical pp scattering, $A_{LS} = A_{SL}$ and near the forward direction one could check the expected equality $A_{NN} \approx A_{SS}$. Like A_{SL}, the spin asymmetry with one of the initial protons polarized has a $\sqrt{-t}$ factor and so in

$$-A_N \mathcal{I}_1 \;=\; \alpha\, a_N \frac{\sigma_{\rm tot}}{8\pi} + b_N\, t + \cdots$$

it is convenient to define another scaled unpolarized differential cross section

$$\mathcal{I}_1 \equiv \frac{m\sqrt{-t}}{e^{2Bt}\sigma_{\rm tot}} \cdot \frac{d\sigma}{dt}$$

The eight expressions for the measurable coefficients, a_j, b_j, of the asymmetry expansions are sufficient to determine

$$R_j,\, I_j, \quad j = +,\, -,\, 2,\, 5$$

or at least provide bounds on the pp amplitudes.

Table 1: The expansion coefficients a_j

Observable	a_j
$\mathcal{I}_0$	ρ
$A_{NN}\mathcal{I}_0$	R_2
$A_{LL}\mathcal{I}_0$	R_-
$-A_{SL}\mathcal{I}_1$	$\kappa(R_2 + R_-)/2$
$-A_N\mathcal{I}_1$	$I_5 - \kappa(1 + I_2)/2$

For $A_N(pp)$, note the occurence of both $I_5 = \text{Im}\ r_5$ and I_2 in coefficient a_N . Amplitude I_2 would be absent in $A_N(pC)$.

Table 2: The expansion coefficients b_j

Observable	b_j
$\mathcal{I}_0$	$(1+\rho^2+R_-^2+I_-^2)/2+R_2^2+I_2^2$
$A_{NN}\mathcal{I}_0$	$R_2(\rho+R_-)+I_2(1+I_-)$
$A_{LL}\mathcal{I}_0$	$\rho R_- + I_- + R_2^2 + I_2^2$
$-A_{SL}\mathcal{I}_1$	$R_5(R_2+R_-)+I_5(I_2+I_-)$
$-A_N\mathcal{I}_1$	$I_5(\rho+R_2)-R_5(1+I_2)$

Analyzing power

- The analyzing power has a maximum in interference region
- Spin effects change the value of the maximum of A_N
- Spin dependent amplitudes can be obtained from double spin asymmetry measurements near forward angles
- A study of the energy dependence of forward helicity amplitudes through the use of analyticity could test causality

Polarization Evolution

When circulating at frequency ν through a polarised target of areal density n and polarisation P normal to the ring plane,

$$\frac{d}{dt}\begin{bmatrix} N \\ J \end{bmatrix} = -n\,\nu \begin{bmatrix} I_{\mathrm{c}} - I_{\mathrm{a}} & P\,(A_{\mathrm{c}} - A_{\mathrm{a}}) \\ P\,(A_{\mathrm{c}} - K_{\mathrm{a}}) & I_{\mathrm{c}} - D_{\mathrm{a}} \end{bmatrix} \begin{bmatrix} N \\ J \end{bmatrix}$$

describes the rate of change of the number of beam particles $N(t)$ and their total spin $J(t)$. The loss of particles from the beam involves the difference of complete (c) and accepted (a) quantities, the first matrix element of the rate matrix,

$$I_{\rm c} - I_{\rm a} = 2\,\pi \int_{\theta_{\rm a}}^{\pi} \frac{d\,\sigma}{d\,\Omega}\, \sin\theta\, d\theta$$

as coefficient, where integration is over all angles beyond $\theta_{\rm a}$, the acceptance angle of the accelerator ring. Another change in the beam, the second matrix element, results from a product of target polarisation P with

$$A_{\rm c} - A_{\rm a} = \pi \int_{\theta_{\rm a}}^{\pi} (A_{\rm NN} + A_{\rm SS}) \frac{d\,\sigma}{d\,\Omega}\, \sin\theta\, d\theta$$

the azimuthal average of the transverse double spin asymmetry integrated over collision angles beyond the acceptance angle.

A change in the total spin J, a contribution to the third matrix element, involves a product of P with the average transverse asymmetry integrated completely (c) over angles θ

$$A_{\,\mathrm{c}} = \pi \int_{\theta_0}^{\pi} (A_{\mathrm{NN}} + A_{\mathrm{SS}}) \frac{d\,\sigma}{d\,\Omega}\, \sin\theta\, d\theta$$

above a minimum angle θ_0 linked to the average distance between charges. Another contribution to the change in J comes from P times the azimuthally averaged spin transfer observable

$$K_{\mathrm{a}} = \pi \int_{\theta_0}^{\theta_{\mathrm{a}}} (K_{\mathrm{NN}} + K_{\mathrm{SS}}) \frac{d\,\sigma}{d\,\Omega}\, \sin\theta\, d\theta$$

integrated over low angles where particles remain in the ring.

A contribution to a change in J involving J itself, part of the fourth element of the rate matrix, results from the azimuthally averaged depolarisation observable, below acceptance

$$D_{\rm a} = \pi \int_{\theta_0}^{\theta_{\rm a}} (D_{\rm NN} + D_{\rm SS}) \frac{d\,\sigma}{d\,\Omega}\, \sin\theta\, d\theta$$

in the form of a loss of polarisation quantity, again integrated over angles within the acceptance range

$$L_{\rm a} = (I_{\rm a} - D_{\rm a})\,/\,2\,.$$

Using determinants to solve the coupled system provides the time dependence of the beam polarisation $P_{\rm b}$ induced by P

$$P_{\rm B}(t) \;=\; \frac{J(t)}{I(t)} \;=\; P \, \frac{K_{\rm a} \,-\, A_{\rm c}}{L_{\rm a} \;+\; L_{\rm d} \coth\left(L_{\rm d}\, n\, \nu\, t\right)}$$

Takakazu Seki (Fujioka 1642–1708 Edo) was the first to study determinants (1683); the discriminant for the eigenvalues is

$$L_{\,\rm d} \;=\; \sqrt{P^{\,2}\,(A_{\rm c} \,-\, A_{\rm a})\,(A_{\rm c} \,-\, K_{\rm a}) \,+\, L_{\rm a}^2}\;.$$

In the short term, the rate of change of polarisation is around

$$\frac{dP_{\rm B}}{dt} \;\approx\; n\,\nu\,P\,(K_{\rm a} \,-\, A_{\rm c})$$

In the long term, the polarisation increases in magnitude to

$$\lim_{t\to\infty} P_{\mathrm{b}}(t) \;=\; P\,\frac{A_{\mathrm{c}} \,-\, K_{\mathrm{a}}}{L_{\mathrm{a}} \,+\, L_{\mathrm{d}}}\,.$$

In the short term, the rate of change of luminosity is around

$$\frac{dI}{dt} \;\approx\; n\,\nu\,P\,(I_{\mathrm{a}} - I_{\mathrm{c}})$$

Such quantities relate to integrals over double spin asymmetries that have singular behaviour in t and are enhanced. Normal and longitudinal polarizations of the source of polarization build-up lead to different rates of change and provide information on the values of ρ, R_2, I_2, R_-, I_-, R_5, and I_5 for the pp case.

Conclusions

- Spin dependent hadronic collisions test nonperturbative QCD
- Maximum analyzing power at interference probes spin effects
- Double spin asymmetries near forward direction show effects
- Kinetics of stored proton polarization provides information
- Causality test via analyticity of spin dependent amplitudes.