On Compensation of Beam Depolarization at Crossing of a Spin Resonance

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Spin Adiabatic Invariant (SAI)

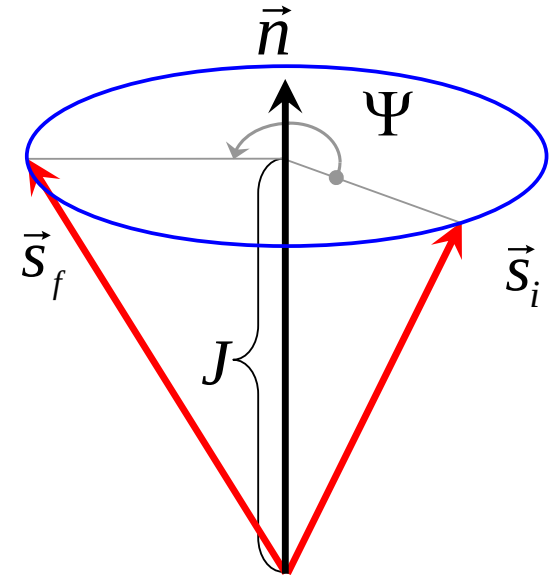
[L.H. Thomas -1927, V. Bargmann, L. Michel, V.L. Telegdi -1959]

$$\frac{d\vec{S}}{d\theta} = [\vec{W} \times \vec{S}], \qquad \vec{W}(I_i, \Psi_i, \theta) = \vec{W}(I_i, \Psi_i + 2\pi, \theta + 2\pi)$$

There is the exactly defining direction of precession axis [Ya.S. Derbenev, A.M. Kondratenko, 1972-1973] :

$$\vec{n}(I_i, \Psi_i, \theta) = \vec{n}(I_i, \Psi_i + 2\pi, \theta + 2\pi)$$

the direction of the quantum axis



(J, Ψ) – act and phase variables of spin motion:

$$J = \vec{S} \cdot \vec{n}, \quad \Psi' = \nu(I_i)$$

$\nu(I_i)$ – generalized frequency of spin precession

The beam polarization degree: $P = \langle J \rangle$

Average spin vector:

$$\langle \vec{S} \rangle = \langle J\, \vec{n}(I_i, \Psi_i, \theta) \rangle = P \langle \vec{n} \rangle$$

J – Spin Adiabatic Invariant (SAI)

Quantum particle with spin ½ : $J = \pm \frac{\hbar}{2}$

Classical particle with spin $|S| \gg \hbar$: $J \in [-S, S]$

Adiabatic spin resonance crossing.

Near in resonance:	Far from resonance:
$\lvert\Delta\vec{n}\rvert \sim 1 \Rightarrow \langle \vec{S} \rangle \to 0$	$\lvert\Delta\vec{n}\rvert \to 0 \Rightarrow \langle \vec{S} \rangle \to P_i\, \vec{n}$
$P_{res} \neq 0 \quad P_{res} = P_i$	$P_f = P_i$

Spin motion near spin resonance

Resonance condition:

$$\nu_0 = \nu_k\,, \qquad \nu_0 = \gamma\frac{g-2}{2} = \gamma G$$

In resonance coordinate system

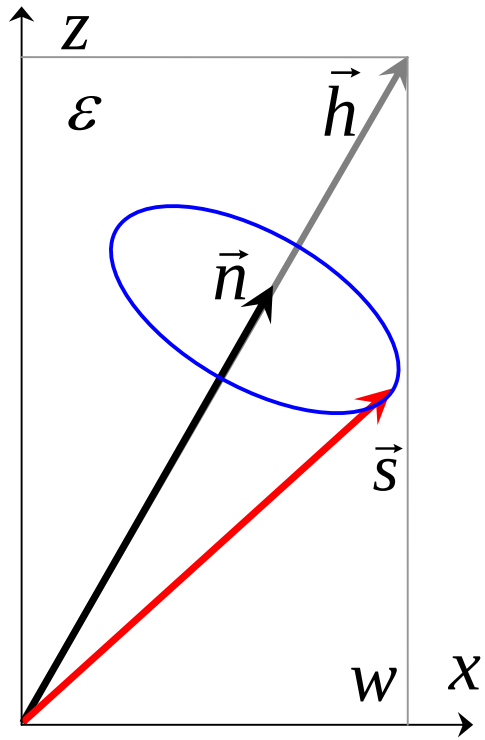
[Ya.S. Derbenev, A.M. Kondratenko, A.N. Skrinsky, 1971] :

$$\vec{h} = \varepsilon\,\vec{e}_z + \vec{w}_k\,, \qquad \varepsilon = \nu_0 - \nu_k\,.$$

$\vec{w}_k -$ resonance strength (Fourier spin perturbation harmonic)

$\varepsilon -$ resonance detune

$$\nu = \sqrt{\varepsilon^2 + w^2}$$



Adiabatic condition: $\left|\vec{h}'\right| \ll h^2$

if $w = const \quad \Rightarrow \quad \left|\varepsilon'\right| \ll \varepsilon^2 + w^2$

Adiabatic resonance cross:

$$\left|\varepsilon'\right| \ll w^2$$

Before and after resonance crossing

$$\vec{n} \to \begin{cases} \vec{e}_z, & \text{if } \nu_0 > \nu_k \\ -\vec{e}_z, & \text{if } \nu_0 < \nu_k \end{cases}$$

One-time isolated resonance crossing $(\varepsilon' = const)$

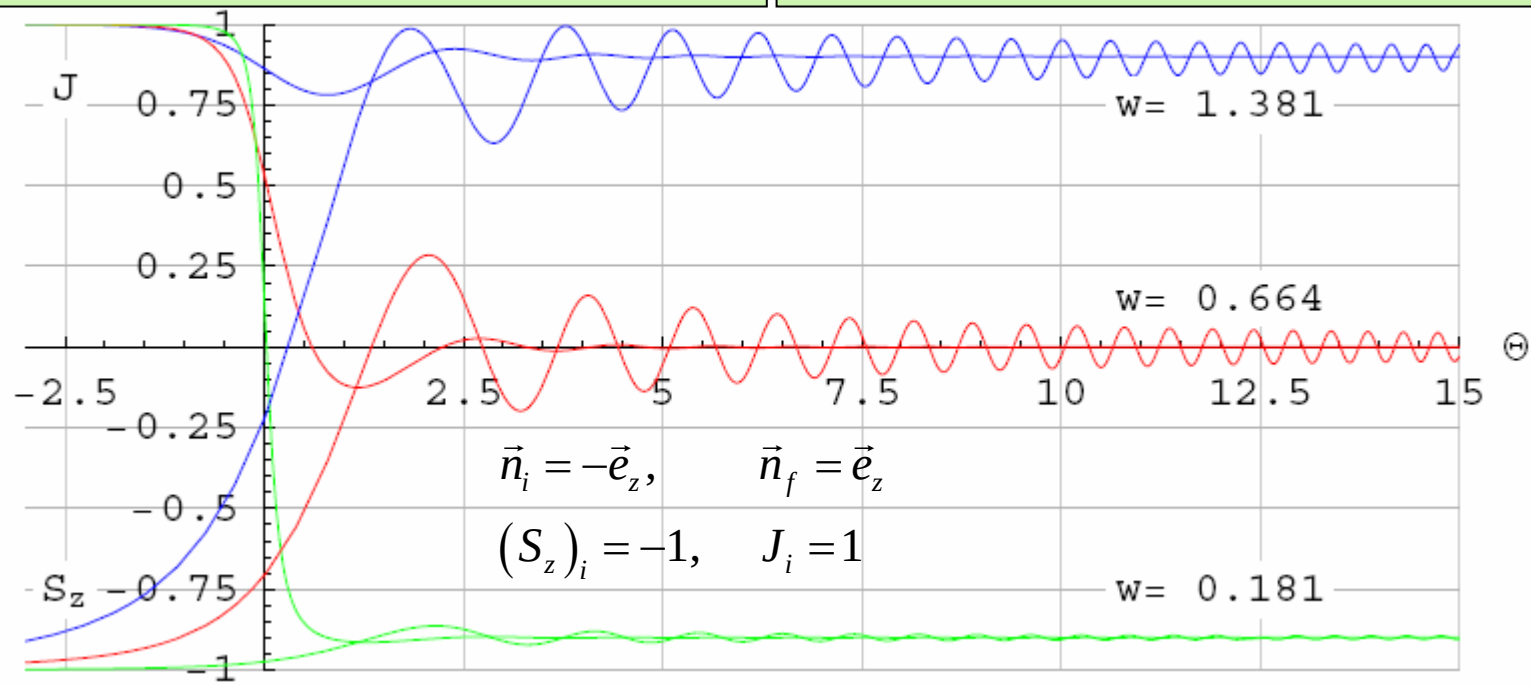
$\mathrm{w} = \dfrac{w}{\sqrt{\varepsilon'}}$ – normalized resonance strength

$\Theta = \sqrt{\varepsilon'}\theta$ – normalized azimuth

$\mathrm{w} \ll 1$ – fast resonance crossing

$\mathrm{w} \sim 1$ – intermediate resonance crossing

$\mathrm{w} \gg 1$ – adiabatic resonance crossing



M. Froissart, R. Stora , 1960

$$\left\langle S_z^f \right\rangle = \left\langle S_z^i \left(\exp\left(-\frac{\pi \mathrm{w}^2}{2} \right) - 1 \right) \right\rangle,$$

$$\Theta_i = -\infty, \qquad \Theta_f = \infty.$$

Changing $\vec{e}_z \to \vec{n}$ $\left(S_z^i \to -J_i,\ S_z^f \to J_f \right)$:

$$P_f = P_i \left(1 - \exp\left(-\frac{\pi \mathrm{w}^2}{2} \right) \right)$$

$$|\Theta_i|, |\Theta_f| >> 1$$

Spin resonance intersection techniques

Various techniques of spin resonances intersection can be based on the following factors:

- increasing the speed of the spin-resonance crossing due to "jump" of the betatron frequency;
- increasing the speed of the spin-resonance crossing due to "jump" of the spin frequency;
- resonance strength compensation;
- a deliberate increase of the spin-resonance strength by means of specially introduced magnetic fields for adiabatic intersection of spin resonance;
- deliberate decrease of the spin-resonance crossing speed.

Preserving polarization degree condition

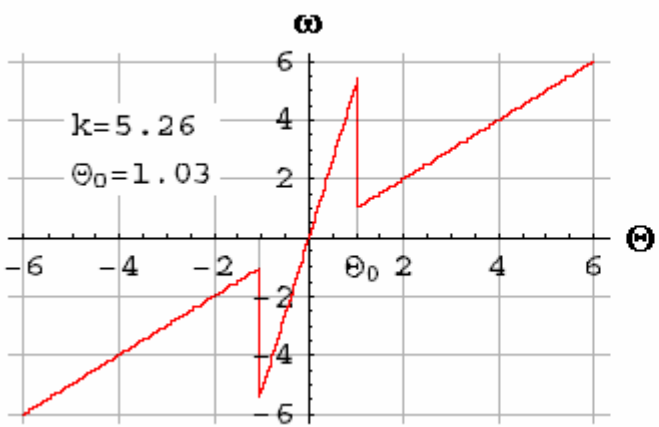
J_i – initial value of SAI , J_i – final value of SAI

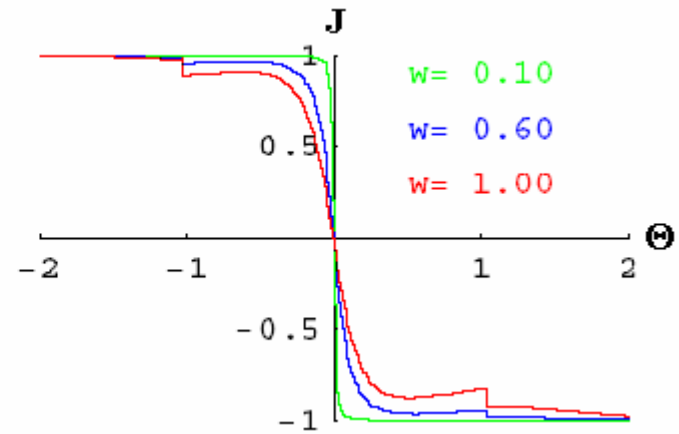
$$J_f = J_i \quad \text{or} \quad J_f = -J_i$$

The example of preserving beam polarization degree $\left(J_f = -J_i\right)$

[A.M. Kondratenko , M.A. Kondratenko, Yu.N. Filatov-2004]

$$\omega(\Theta) = \begin{cases} \Theta & \text{if } |\Theta| \geq \Theta_0 \\ k\Theta & \text{if } |\Theta| < \Theta_0 \end{cases}, \quad \text{where } \omega(\Theta) = \frac{\varepsilon(\theta)}{\sqrt{\varepsilon_0'}},\ k = \frac{\varepsilon_1'}{\varepsilon_0'},\ \mathrm{w} = \frac{w}{\sqrt{\varepsilon_0'}}$$





Fast crossing compensation condition:

$$\int_{-\infty}^{\infty} \exp\left(\int_0^{\theta} \varepsilon(\theta)\, d\theta\right) d\theta = 0$$

The polarization compensation will occur with high accuracy as w^6

Shift resonance point at $\Delta\omega < 0.48 \Rightarrow \left(D < 0.01 D_0\right)$

The example of two-time isolated resonance crossing

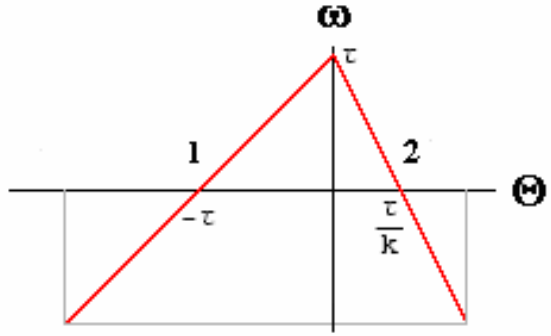
Dependence of normalized detuning from time (normalized azimuth):

$$\omega(\Theta) = \varepsilon(\theta)/\sqrt{\varepsilon_1'},$$

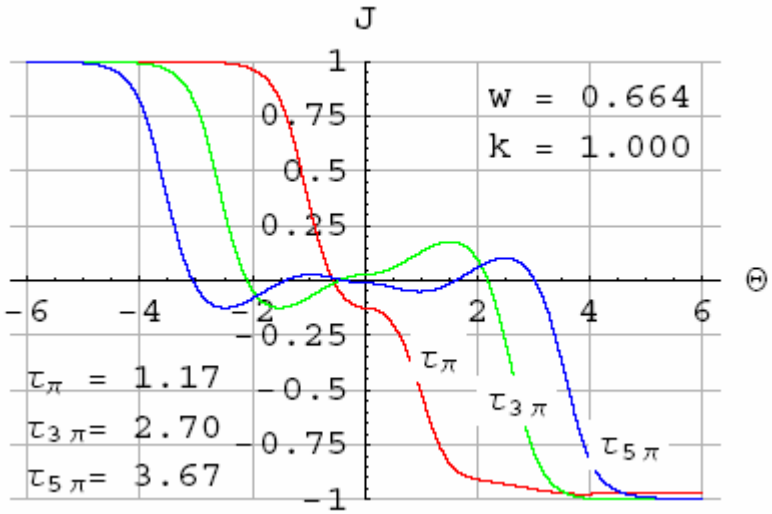
$$\omega(\Theta) = \begin{cases} \tau - k\Theta & \text{if } \Theta \geq 0 \\ \Theta + \tau & \text{if } \Theta < 0 \end{cases},$$

where

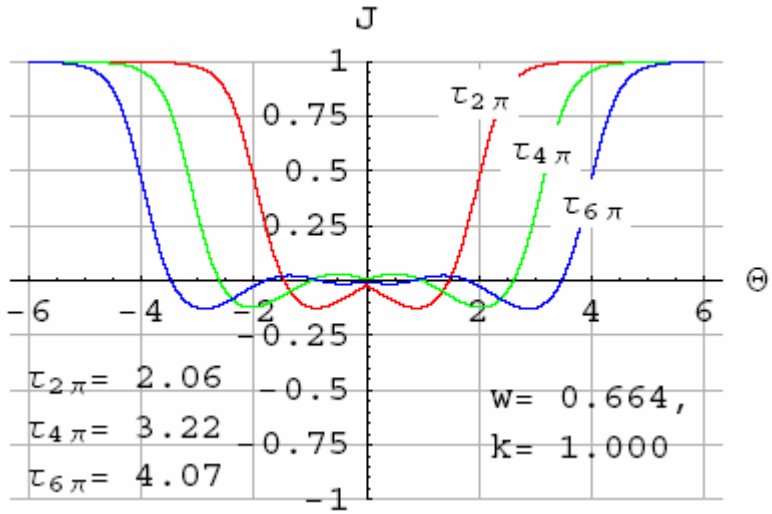
$$\Theta = \theta\sqrt{\varepsilon_1'}, \quad k = -\frac{\varepsilon_2'}{\varepsilon_1'}, \quad \tau = \theta_1\sqrt{\varepsilon_1'}.$$



Normalized detuning vs normalized time



SAI vs time



SAI vs time

The example of three-time isolated resonance crossing

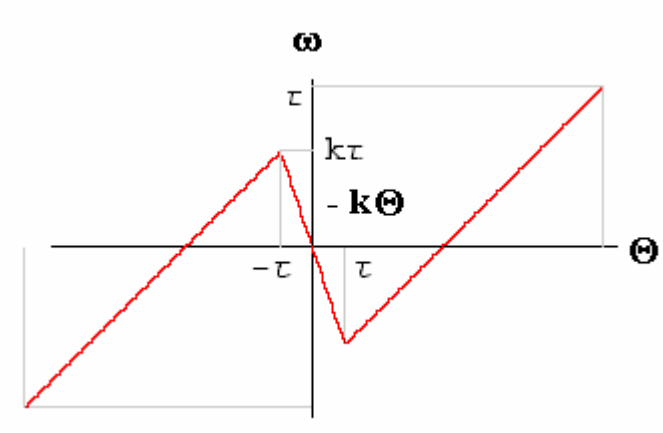
Dependence of normalized detuning from time (normalized azimuth):

$$\varepsilon(\theta) = \omega(\Theta)\sqrt{\varepsilon_1'},$$

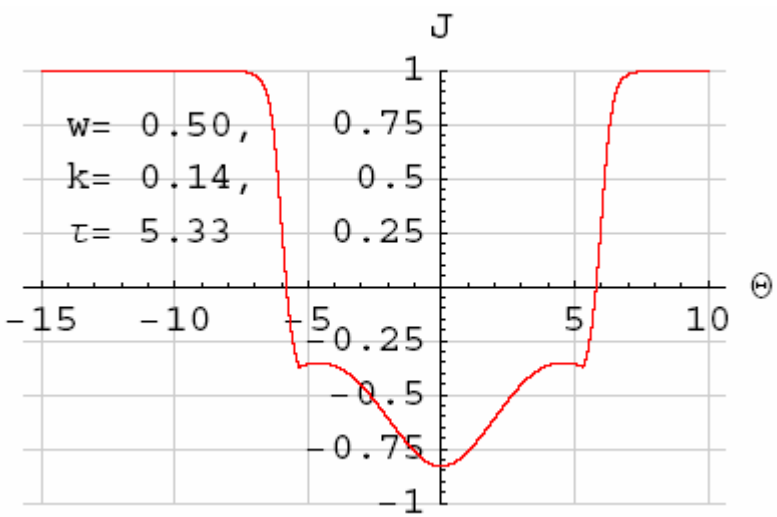
$$\omega(\Theta) = \begin{cases} -k\Theta, & \text{if } |\Theta| \le \tau \\ \Theta - \mathrm{sign}(\Theta)(k+1)\tau, & \text{if } |\Theta| > \tau \end{cases}$$

where

$$\Theta = \theta\sqrt{\varepsilon_1'}, \quad k = \frac{\varepsilon_2'}{\varepsilon_1'} > 0$$



Normalized detuning



The dependence of SAI versus time

Spin detuning spread which is reduce polarization degree at 10%

$$\frac{\Delta\varepsilon_{10\%}}{\varepsilon(\tau)} \le 0.53$$

Polarization degree is preserved due to interference between resonances.

Where are no increasing resonance crossing rate

Conclusions

- The method of preservation of beam polarization at crossing of spin resonance in cyclic accelerators is offered.
- This method is based on control the spin precession axis and the spin rotation phase inside of the resonance region.
- An analytical solution for an arbitrary resonance crossing speed is presented.
- The method to operate by a vector of polarization in a ring of the accelerator giving additional opportunities to provide experiments with internal and extract beams is offered.
- The numerical examples are presented.

Basic Nuclotron Parameters

Charge to mass ratio ions		0.33 – 0.5, 1
Injection energy	nuclei protons	5 MeV/Amu 20 Mev
Maximum energy	for nuclei with q/A=0.5 for protons	6 GeV/Amu 12.8 GeV
Transition energy		7.6 GeV
Circumference		251.52 m
Duration of acceleration		1 sec
Magnetic rigidity	at injection maximum	0.647 T·m 45.83 T·m
Betatron tunes	ν_x ν_z	6.8 6.85
Normalized emittance at injection		$45\pi \times$ mm $\times$ mrad

	^{1}H	^{2}H	^{3}H	^{3}He
$E_k^{\max}, \frac{\text{GeV}}{\text{n}}$	12.85	6.00	3.74	8.28
$G = \frac{g-2}{2}$	1.793	-0.143	7.92	–4.184
Number of intrinsic resonance $\nu = k\,P \pm \nu_z$	6	–	8	9
Number of integer resonance $\nu = k$	25	1	32	37

Intrinsic spin resonance in Nuclotron (p, T, He-3)

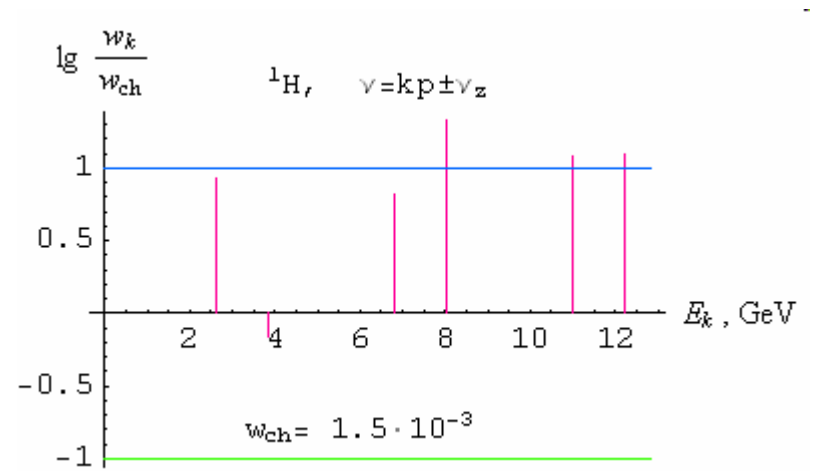
$$w_k = \frac{1}{2\pi}\int_0^{2\pi} w_\perp^* e^{i\nu_k \tilde{K}_z}\, d\theta,$$

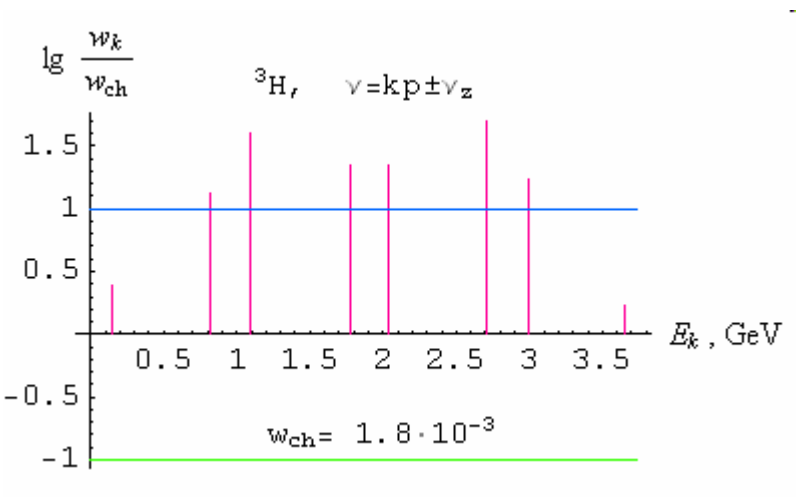
$$w_{ch} = \sqrt{\frac{\varepsilon'}{\pi}}$$ (beam depolarization)

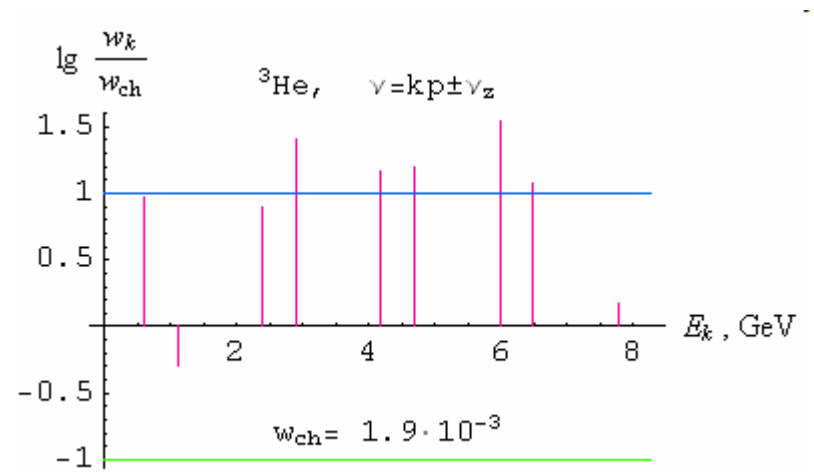
Above blue line – adiabatic crossing

Under green line – fast crossing

Between lines – intermediate crossing







Integer spin resonance in Nuclotron (p, T, He-3)