STUDY OF POSSIBILITIES FOR A SPIN FLIP IN HIGH ENERGY ELECTRON STORAGE RING HERA

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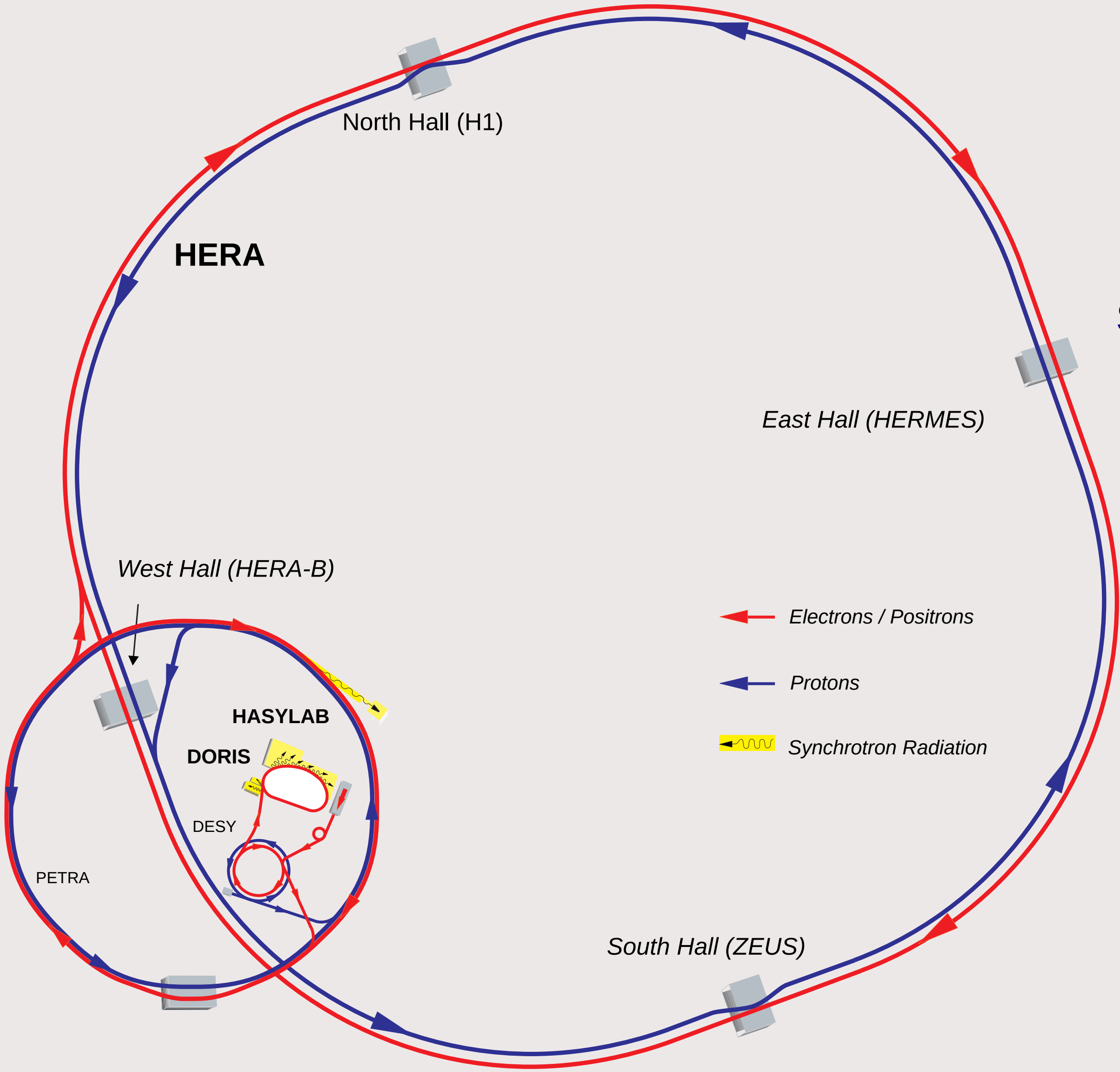
Kyoto, 2.10.2006
SPIN 2006

- Introduction
- Motivation
- Spin tune and decoherence of particle spins
- Model
  - Differential equations of spin motion
  - Differential equations of orbital motion
- Implementation (Fokker-Planck : Monte Carlo?)
- Model predictions: Spin/Polarization reversal - is it possible, to what extent and why?
- Summary

INTRODUCTION



Storage ring HERA, DESY

- L=6300m
- two beams
  - E(e^+/e^-)=27.5GeV
  - E(p)=920GeV

(HERMES, (H1, ZEUS) - usage of polarized electron beams - spin rotators)

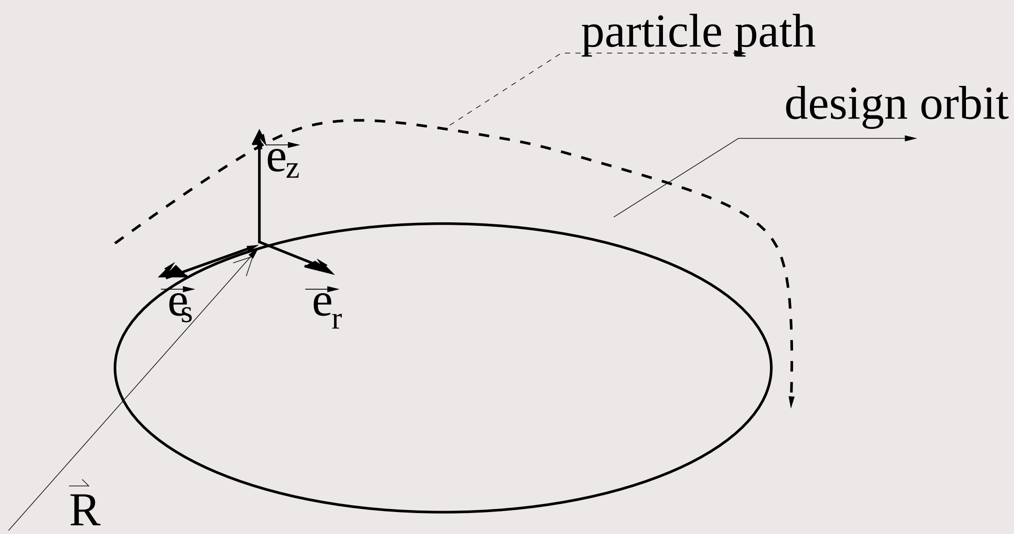
Particle motion in storage ring is described with Lorenz force

$$e\vec{E}_e + \frac{e}{c}\dot{\vec{r}} \times \vec{B} + \vec{\mathcal{R}} = \frac{d}{dt}\frac{E}{c^2}\dot{\vec{r}} \quad , \qquad \dot{\vec{r}} = \frac{d\vec{r}}{dt} \quad . \tag{1}$$

$\vec{E}_e$, $\vec{B}$ - EM fields, E particle energy, $\vec{\mathcal{R}}$ sinchrotron radiation emission force and $\vec{r}$ particle position in the ring.

synchronous particle - reference orbit $\vec{R}$: gains and looses same amount of energy in one turn

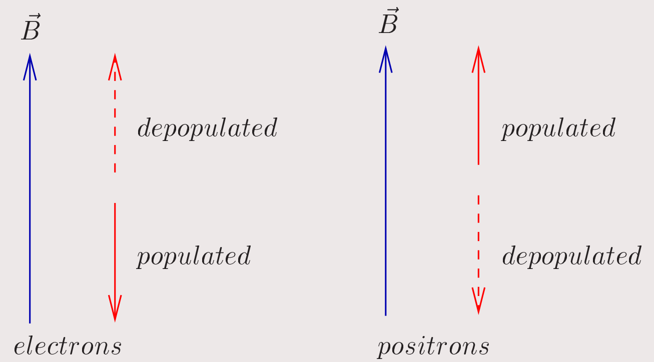
Expansion around $\vec{R}$: $\vec{r} = \vec{R} + (s, r, z)$
$\longrightarrow$ non-linear system 6 differential equations of the first order

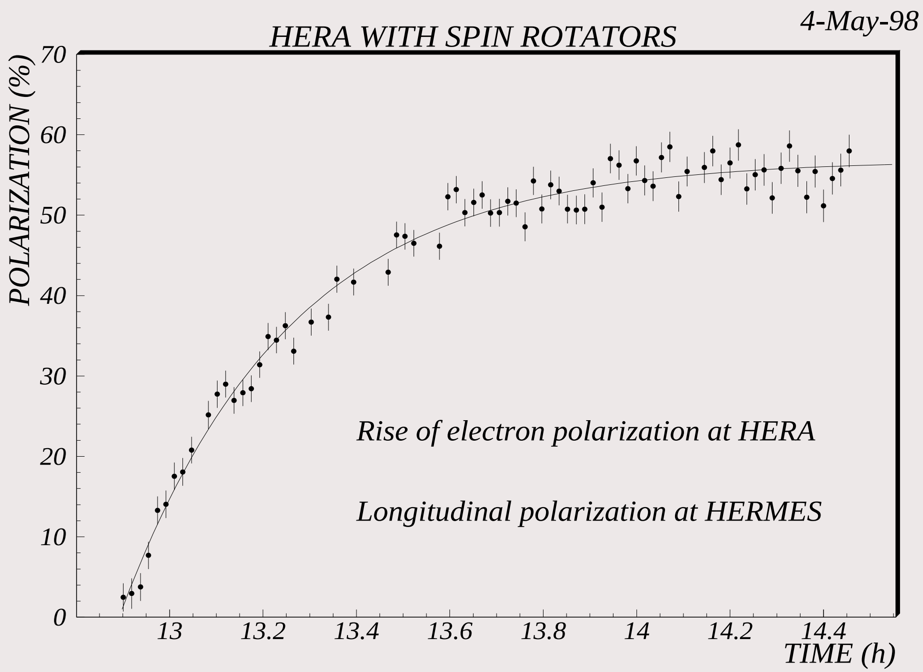


- 2 equations for longitudinal motion (synchrotron motion - bunch: σ, η)
- 4 equations for transverse motion (betatron motion: $r, dr/ds, z, dz/ds$)

MOTIVATION

- electron and positron spins in high energy storage rings become spontanously polarized via the emission of spin-flip synchrotron radiation (Sokolov-Ternov).





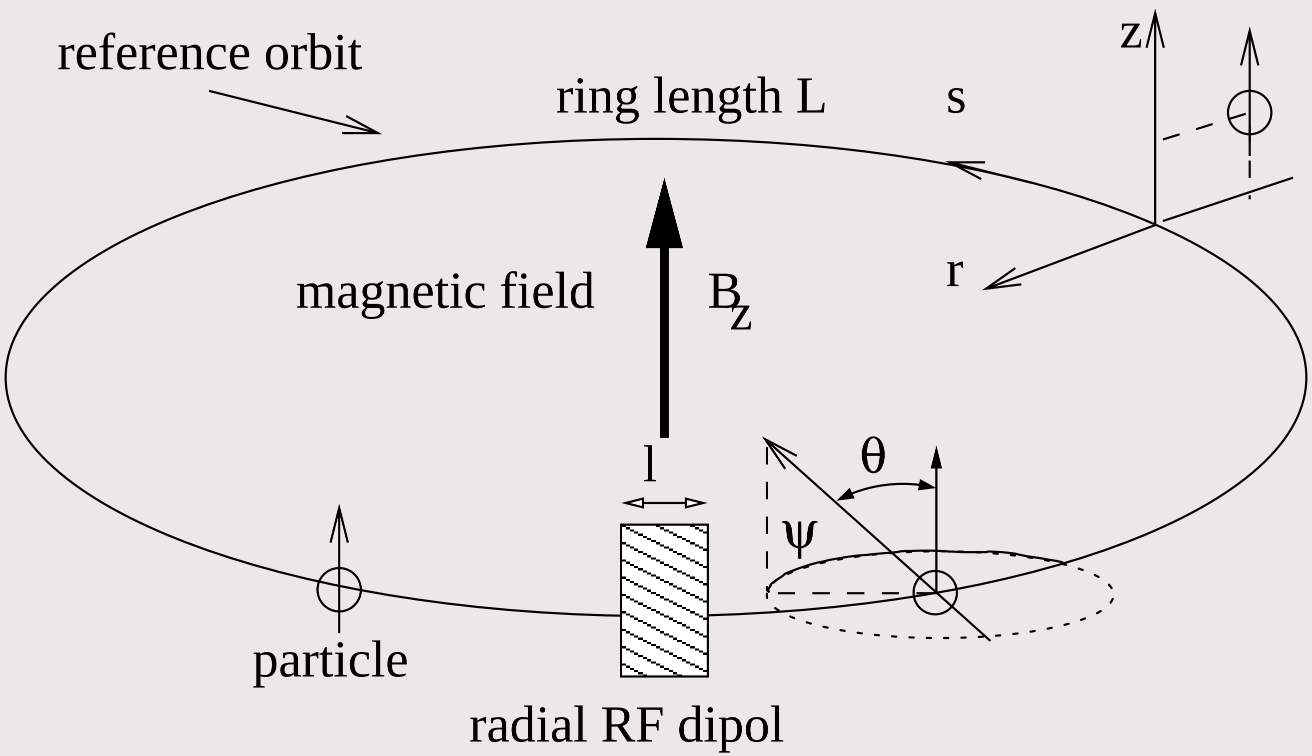
1. EXPERIMENTAL BENEFIT?

- polarized $\vec{S}$ ➡ "clean experiments"
- change direction during operation - during the same run
- flipping spins ➡ reduce systematic errors in spin assymetry measurements

2. Spin flip experimentally obtained

- theoretically not completely explained (2D models predict: complete depolarization!)

Spin manipulation?



localized radio-frequency (RF) dipole kicker (radial disturbing magnetic field)

$$\vec{B}_{RF}(t) = B_{RF}\vec{e}_r = B^0_{RF}\ \cos\left(2\pi f_{RF} t\right)\vec{e}_r \quad .$$

applied on section $l \ll$ ring length L.

- resonant spin flipping: employing a RF dipole kicker (NMR analogy)
- ramping the frequency of the disturbing magnetic field through the resonance (speed of freq.changing)
  - resonance strenght $\epsilon(B_{RF})$ and frequency ramping rate α ($f_{RF} = f_0 + \alpha t$)

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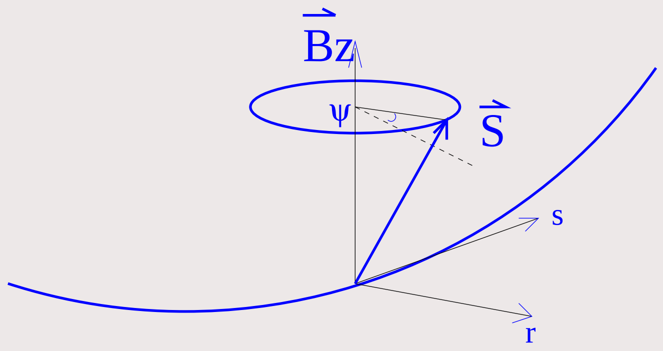
ORIGIN OF SPIN PRECESSION SPREAD-DECOHERENCE



SPIN PRECESSION RATE, SPIN TUNE

- (in B_z) spins precess around the vert. axis

$$S_z = \cos\Theta \approx const$$
$$S_{horz} = S_s + iS_r \approx e^{i\nu\Psi}$$

- spin precession rate - number of spin precessions per turn $\nu = a\gamma$
- spin tune - for a synchronous particle $\nu_0 = a\gamma_0$
- NOTICE: depends on particle energy (SR, synhrotron/betatron motion)
  - energy spread $\Rightarrow$ spin precession rate spread & spin precession decoherence

IMPORTANT: B_{RF} too weak to flip spins in one turn (required $\int B_{RF} dl$ too big)

- rotation for π with $\sum$ small spin rotations after many turns - long term procedure
- for the B_{RF} partial spin rotations to add cumulatively (resonance), f_{RF} must match ν (or $\bar{\nu}$ - non integer part of spin tune):

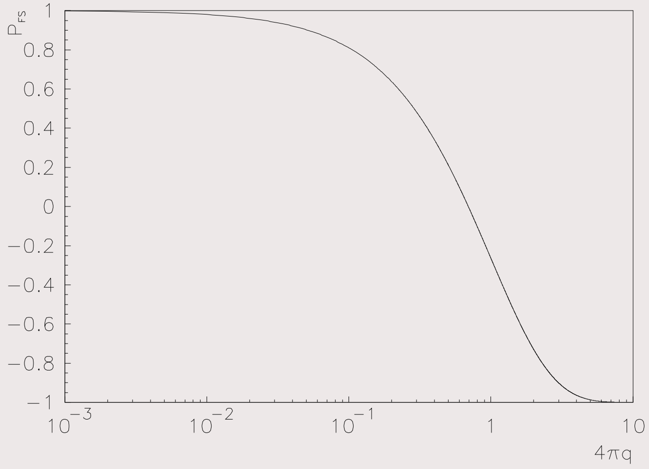
$$f_{RF} = f_c\bar{\nu} \text{ or } f_{RF} = f_c(1-\bar{\nu}) \text{ (mirror values)}$$

frequency of the RF dipole field

circulation freq.e^-

ENERGY SPREAD AND LONG TERM PROCEDURE MAKE SPIN FLIPPING DIFFICULT!

Spin manipulation: polarization direction under B_{RF} - ideal case



FROISSART-STORA EQUATION

$$\frac{P_z^f}{P_z^i} = 2e^{\frac{-\pi|\epsilon|^2}{2\alpha}} - 1$$

$4\pi q$

not including:

- synchrotron radiation (SR)
- synchrotron/betatron (!) motion

- before crossing the resonance: initial $P_z^i \Rightarrow$ after: final P_z^f - *vertical polarization* = P_{FS}
- P_z^f as a function of
  - resonance strength $\epsilon(B_{RF})$ and
  - speed of crossing the resonance α - ramping rate

to describe nonideal cases

we MUST include:

- synchrotron radiation (SR)
- synchrotron/betatron (!) motion

MODEL - Differential equations of spin motion

For each particle i:

- $\boxed{\vec{\Omega}_i} = (\Omega_{ix}, \Omega_{ir}, \Omega_{iz})$, precession vector
- unit spin vector $\vec{S}_i$, semiclassical description in spherical coordinates

$$\vec{S}_i = \frac{\hbar}{2} \begin{pmatrix} \cos\Psi_i \sin\Theta_i \\ \sin\Psi_i \sin\Theta_i \\ \cos\Theta_i \end{pmatrix}$$

Ψ - spin precesssion angle

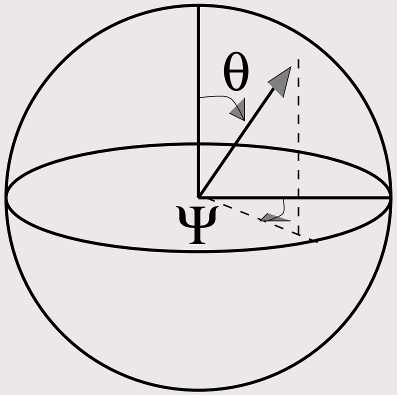
Θ - vertical spin angle

Using precession equation $\boxed{\frac{d\vec{S}_i}{ds} = \vec{\Omega}_i \times \vec{S}_i}$ (for highly relativistic particles $v \approx c \Rightarrow s = ct$)

we obtain

$$\frac{d\Theta_i}{ds} = \Omega_{ir} \cos\Psi_i - \Omega_{ix} \sin\Psi_i$$

$$\frac{d\Psi_i}{ds} = \Omega_{ir} \sin\Psi_i \frac{\cos\Theta_i}{\sin\Theta_i} - \Omega_{ix} \cos\Psi_i \frac{\cos\Theta_i}{\sin\Theta_i} + \Omega_{iz}.$$



To describe spin motion, parameters $\vec{\Omega}_i$ for a given storage ring must be found ...

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… a model of spin dynamics in 3D

Thomas-BMT equation (EM fields in laboratory system)

$$\vec{\Omega} = \frac{e}{mc\gamma}\left[(1+\gamma a)\vec{B}_\perp + (1+a)\vec{B}_\parallel - (a\gamma + \frac{\gamma}{1+\gamma})\vec{\beta}\times\frac{\vec{E}}{c}\right]$$

1. smooth planar ring approximation no vertical bends, $\vec{B}_\parallel = 0$, $\vec{E} = 0$ $\longrightarrow$

$$\vec{\Omega} = \frac{e}{mc\gamma}\left[(1+\gamma a)\vec{B}_\perp\right]$$

2. spin tune for a particle with relative energy offset η $\nu = a\gamma = a\gamma_0(1+\eta)$ $\longrightarrow$

$$\vec{\Omega} = \frac{e}{mc\gamma}\left[(1+a\gamma_0(1+\eta))\vec{B}_\perp\right]$$

3. with $\vec{B}_\perp = (0, B_{RF}F_l(s), B_z)$, $B_{RF} = B_{RF}^0\cos(2\pi f_{RF}s/c)$ - RF disturbing field $\longrightarrow$

$$\vec{\Omega} = \frac{B_z e}{mc\gamma}\left[1+a\gamma_0(1+\eta)\right]\vec{e}_z + \frac{B_{RF}e}{mc\gamma}\left[1+a\gamma_0(1+\eta)\right]F_l(s)\vec{e}_r$$

where $F_l(s) = \sum_{k=0}^{\infty}\left[H(s-kL) - H(s-l-kL)\right]$
$H(s)$ ... Heaviside step function on the section $[0,l]$, $l \ll L$ and k int

4. in coord.syst. $(\vec{e}_s, \vec{e}_r, \vec{e}_z)$ $\longrightarrow$ $\vec{\Omega} = \frac{B_z e}{mc\gamma}\left[a\gamma_0(1+\eta)\right]\vec{e}_z + \frac{B_{RF}e}{mc\gamma}\left[1+a\gamma_0(1+\eta)\right]F_l(s)\vec{e}_r$

5. $Q_s \ll \nu$ (sinchr. osc. slower than spin precession): average $\vec{\Omega}$ around the ring $\longrightarrow$ $\vec{\Omega} = D(1+\eta)\vec{e}_z + \Omega_r^0\vec{e}_r$

with parameters $D = \frac{2\pi\nu_0}{L}$ $\Omega_r^0 = (1+a\gamma_0)\frac{e}{mc\gamma_0}B_{RF}^0\cos(2\pi f_{RF}s/c)F_l(s)$

Why synchrotron motion canNOT be neglected?

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Spin precession in horiz.plane for an equilibrium beam (SR significant) and without disturbing RF field.

Spin precession rate: $\nu = a\gamma = \nu_0\,(1+\eta) = \nu_0(1+\eta_0\cos\psi_s), \quad \psi_s = Q_s\theta, \quad \theta = 2\pi s/L$

$\longrightarrow$ Q_s synchrotron tune, ψ_s synchrotron phase, η_0 synchrotron osc.amplitude

Therefore spin phase:

$$\Psi = \int_0^\theta \nu d\theta' = \nu_0\theta + \nu_0\frac{\eta_0}{Q_s}\sin\psi_s$$

and horizontal spin component - with expanding into series

$$S_{horz} \sim e^{i\Psi} = e^{i\nu_0\theta}e^{i\nu_0(\eta_0/Q_s)\sin\psi_s} = e^{i\nu_0\theta}\sum_{m=-\infty}^{\infty} e^{im\psi_s}J_m\left(\frac{\eta_0\nu_0}{Q_s}\right)$$

J_m is a Bessel function. After uniform

- averaging over all synchrotron phases ψ_s, only a term $m=0$ remains:

$$\langle S_{horz}\rangle \sim e^{i\nu_0\theta}J_0\left(\frac{\eta_0\nu_0}{Q_s}\right)$$

- averaging over η_0 amplitudes with Gaussian distribution

$$\langle\langle S_{horz}\rangle\rangle \sim e^{i\nu_0\theta}\int_0^\infty J_0\left(\frac{\eta_0\nu_0}{Q_s}\right)e^{-\eta_0^2/(2\epsilon_s)}\frac{d\eta_0}{\sqrt{2\pi\epsilon_s}},$$ ϵ_s-emittance

$$\sim e^{i\nu_0\theta}e^{-\sigma_s^2}I_0(\sigma_s^2) = e^{i\nu_0\theta}\ \text{spin spread}(\sigma_s^2)$$

Width of spin spread due to synchrotron osc. $\sigma_s^2 = \epsilon_s\frac{\nu_0^2}{4Q_s^2}$

LET's EVALUATE - for SYNCHROTRON MOTION:

spin spread$(\sigma_s^2) = e^{-\sigma_s^2} I_0(\sigma_s^2)$

width of spin spread $\sigma_s^2 = \epsilon_s \frac{\nu_0^2}{4Q_s^2}$

typical values for HERA $\eta_0 = 0.001, \nu_0 = 60, Q_s = 0.06$:

$(\epsilon_s = \sigma_{\eta_0}^2)$

$\sigma_s^2 = \epsilon_s \frac{\nu_0^2}{4Q_s^2}$ $\rightarrow 1$ *

$\Rightarrow$ synhr. motion canNOT be neglected

VALID IF:

- if synchr. motion completes many periodes during the time of spin reversal,
- if orbital damping and noise exhibit equilibrium in the beam during this same time
- if therefore time, needed for polarization reversal is much longer than synhr. periode Q_s and damping time

* in agreement with K. Heinemann, DESY Report 97-166, 1997

Why betatron motion can be neglected?

1. vertical betatron oscillations: neglected
- in a planar electron ring the vertical betatron emittance is negligible

2. horizontal betatron oscillations: cause the particles to see an additional vert.magn.field,
- betatron stability condition requires non zero field gradient

- assume a constant quadrupole field gradient g
- the quadrupole field of the shape $B_{quad} = gA_r \cos\psi_r, \quad \psi_r = Q_r\theta, \quad \theta = 2\pi s/L$

  $\longrightarrow$ A_r - betatron amplitude, ψ_r betatron phase, Q_r betatron tune.

The additional precession vector in the T-BMT due to betatron motion now is

$$\omega_z^\beta = \frac{e}{m_e c\gamma}(1+a\gamma_0)gA_r\cos\psi_r \approx \frac{e}{m_e c\gamma}a\gamma_0 A_r \cos\psi_r, \quad a\gamma_0 \gg 1.$$

Analogous to the synchrotron case the spin precession rate can be written as

$$\nu = \nu_0(1 + A_r\cos\psi_r)$$

We repeat corresponding calculations and after averaging over betatron phases ψ_r and over different betatron amplitudes A_r we get

$$\langle\langle S_{horz}\rangle\rangle \sim e^{i\nu_0\theta}\, e^{-\sigma_r^2} I_0(\sigma_r^2) = e^{i\nu_0\theta} \quad \text{spin spread}(\sigma_r^2)$$

Width of spin spread due to betatron osc. $\sigma_r^2 = \epsilon_r \frac{\nu_0}{4Q_r^2}$

LET's EVALUATE - for BETATRON MOTION:

spin spread(σ_r^2) $= e^{-\sigma_r^2} I_0(\sigma_r^2)$

width of spin spread $\sigma_r^2 = \epsilon_r \frac{\nu_0^2}{4Q_r^2}$

since typical values for HERA $\epsilon_r = 10^{-8}, \nu_0 = 60, Q_r \approx 100$

$\sigma_r^2 = \epsilon_r \frac{\nu_0^2}{4Q_r^2} \approx 10^{-12}$ $\longrightarrow 0$

$\Rightarrow$ betatron motion CAN be neglected

VALID IF:

- if betatron. motion completes many periodes during the time of spin reversal,
- if orbital damping and noise exhibit equilibrium in the beam during this same time
- if therefore time, needed for polarization reversal is much longer than betatr.periode Q_r and damping time

... a model of spin dynamics in 3D

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   with parameters $D = \frac{2\pi\nu_0}{L}$ $\Omega^0_r = (1+a\gamma_0)\frac{e}{mc\gamma_0}B^0_{RF}\cos(2\pi f_{RF}s/c)F_l(s)$

... a model of spin dynamics in 3D - Resume

... we obtain precession vector of the model

$$\vec{\Omega} = D(1+\eta)\vec{e}_z + \Omega_r^0(s)\vec{e}_r$$

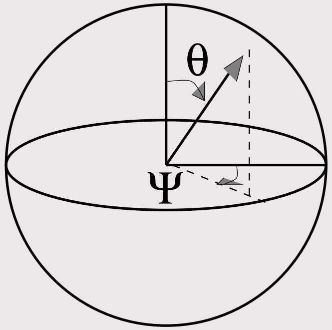
with parameters $D = \frac{2\pi\nu_0}{L}$ $\Omega_r^0 = (1+\nu_0)\frac{e}{mc\gamma_0}B_{RF}^0 \cos(2\pi f_{RF} s/c)$,

η relative energy offset from the synchronous particle

we obtain

$$\frac{d\Theta}{ds} = +\Omega_r^0 \cos\Psi$$

$$\frac{d\Psi}{ds} = -\Omega_r^0 \sin\Psi \frac{\cos\Theta}{\sin\Theta} + D(1+\eta)$$



- Contributions to spin angle change - dΘ:
  - main contribution - resonance term $\propto \Omega_r^0$(s)
  - coupling to synhrotron motion and SR through η

Since $\eta = \eta(s)$ and $\Omega_r^0 = \Omega_r^0(s)$: orbital motion description is needed

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Differential equation of orbital motion

Obtained starting from full set of equations of motion (6D).

- canonical transformation to decouple synchrotron and betatron part *
- averaged over one turn $L \Rightarrow$ betatron motion can be averaged out

(fast compared to spin motion)

$$\frac{d\sigma}{ds} = -\alpha_c \, \eta$$

$$\frac{d\eta}{ds} = \frac{(2\pi \frac{Q_s}{L})^2}{\alpha_c} \frac{L}{2\pi h} \left[\sin(h \, \frac{2\pi}{L} \, \sigma) \; \cos \Phi_0\right] - 2 \frac{\alpha_s}{L} \, \eta + \sqrt{\kappa} \, \xi(t)$$

synhr. osc.

continuous part of SR en.losses

damping effect

stohastic part of SR en.losses

compensates damping - noise

$\Rightarrow$ equillibr.synchr.beam phase space dim.

- here σ - distance from the synchronous particle, η - relative energy offset,
- α_c compaction factor, Q_s synchrotron tune, α_s one turn damping rate, h harmonic number, Φ_0 synchronous particle phase, κ synchrotron radiation excitation strength, $\xi(t)$ Gaussian random variable.

* D.P. Barber et al., DESY Report HERA M-94-09, 1994

Summary of used approximations

approximations:

- highly relativistic particles $v \approx c \Rightarrow s = ct$
- a constant vertical dipol magnetic field $\vec{B}_z$ is applied to the ring (smooth and planar approximation)
- orbital motion averaged around the ring

neglecting:

- nonlinear orbital motion, influence of higher multipoles
- betatron motion:
  - in a planar ring vertical betatron emittance is negligible, therefore vertical betatron oscillations are neglected
  - we prove that the effect of horizontal betatron oscillations can be neglected

Let's calculate ...

IMPLEMENTATION

Final set of differential equations:

- orbital part:

$$\frac{d\sigma}{ds} = -\alpha_c \, \eta$$

$$\frac{d\eta}{ds} = \frac{(2\pi \frac{Q_s}{L})^2}{\alpha_c} \frac{L}{2\pi h} \left[\sin(h \, \frac{2\pi}{L} \, \sigma) \;\; \cos \Phi_0\right] - 2 \frac{\alpha_s}{L} \, \eta + \sum_{j=0}^{\infty} \delta(s - L \frac{j}{N_e}) \sqrt{\kappa_{\mathrm{MC}}} \, \xi(s)$$

- spin part:

$$\frac{d\Theta}{ds} = +\Omega_r^0 \, \cos\Psi$$

$$\frac{d\Psi}{ds} = -\Omega_r^0 \, \sin\Psi \, \frac{\cos\Theta}{\sin\Theta} + D(1+\eta)$$

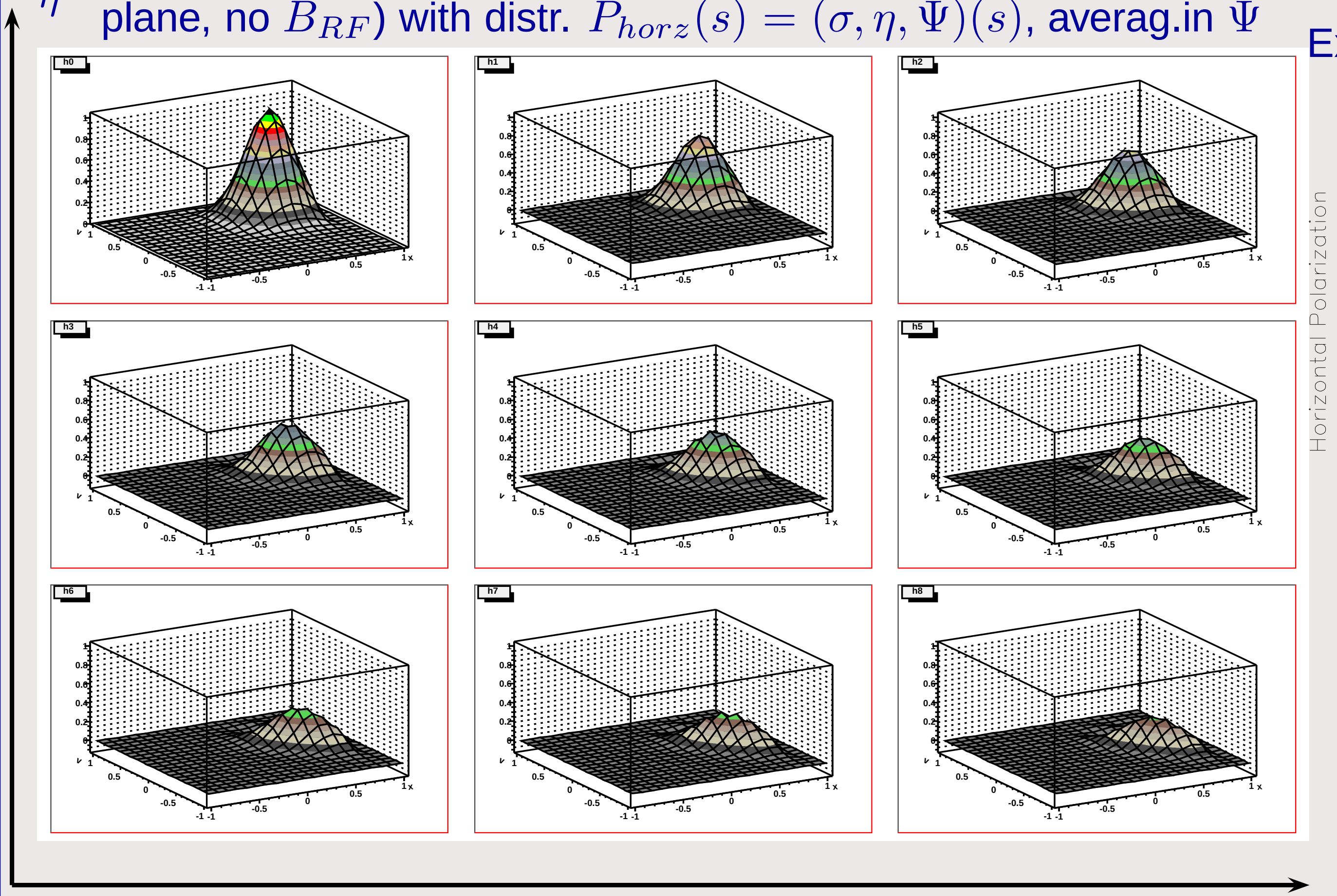
This is a 3D spin problem - possible reduction to a planar 2D spin problem with $\Theta = \pi/2$ (P_{horz}).

Two approaches to solving:

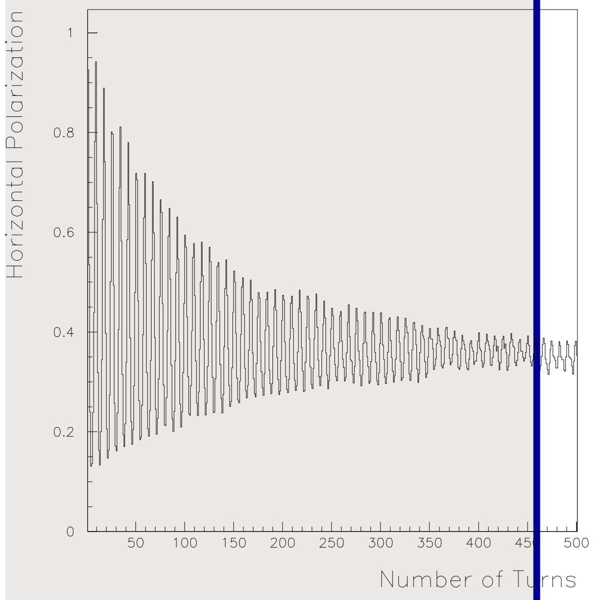
- Monte Carlo: tracking (one by one) particles with phase vector $\vec{Y}(s) = (\sigma, \eta, \Theta, \Psi)(s)$, distribution is obtained through many particles
  - a farm of 300 Pentium III 400 MHz Monte Carlo Array Processors (MAP), University of Liverpool was used - in year 2000
  - a farm of 30 Dual Core Opteron 1.9 MHz after
- Fokker - Planck: distribution $u(\vec{Y}(s))$ on a grid is tracked
  - solving PDE on a parallel computer CRAY T3E, Jülich - in year 1999

Fokker Planck

Example of time development for a 2D spin problem distr. (horiz. plane, no B_{RF}) with distr. $P_{horz}(s) = (\sigma, \eta, \Psi)(s)$, averag.in Ψ



Example of development * $P_{horz}(s) \rightarrow$ limit value



For a 3D spin problem with $\vec{P}(s) = P(\sigma, \eta, \Psi, \Theta)(s)$ calculation is time consuming $\rightarrow$ Monte Carlo.

* in agreement with K. Heinemann, DESY Report 97-166, 1997

Monte Carlo

$$\frac{d\sigma}{ds} = -\alpha_c \, \eta$$

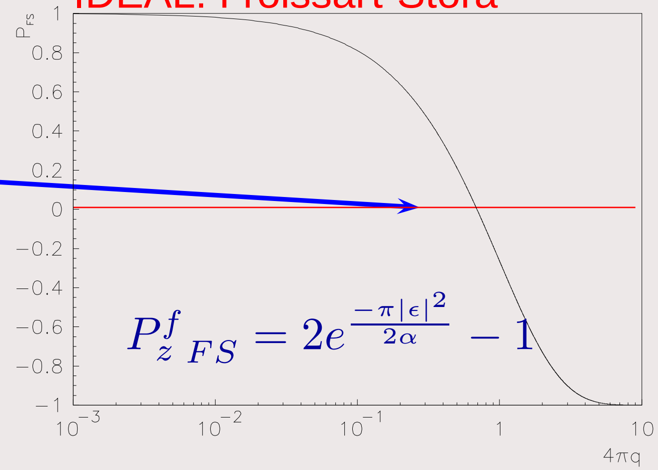
$$\frac{d\eta}{ds} = \frac{(2\pi \frac{Q_s}{L})^2}{\alpha_c} \frac{L}{2\pi h} \left[\sin(h \, \frac{2\pi}{L} \, \sigma) \; \cos \Phi_0\right] - $$
$$- 2 \frac{\alpha_s}{L} \, \eta + \sum_{j=0}^{\infty} \delta(s - L \frac{j}{N_e}) \sqrt{\kappa_{\rm MC}} \, \xi(s)$$

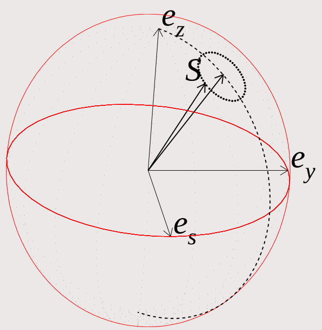
$$\frac{d\Theta}{ds} = +\Omega_r^0 \, \cos\Psi$$

$$\frac{d\Psi}{ds} = -\Omega_r^0 \, \sin\Psi \, \frac{\cos\Theta}{\sin\Theta} + D(1+\eta)$$

- ensemble of N spin $\frac{1}{2}$ particles - electrones
- solve equations numerically for each spin particle i
- ramp frequency of RF field through the resonance
- FIND POLARIZ.DIRECTION AFTER RAMPING:
  $\vec{P}(\epsilon, \alpha) = \frac{1}{N} \sum_{i=1}^{N} \frac{\vec{S}_i}{|\vec{S}_i|}$

IDEAL: Froissart-Stora



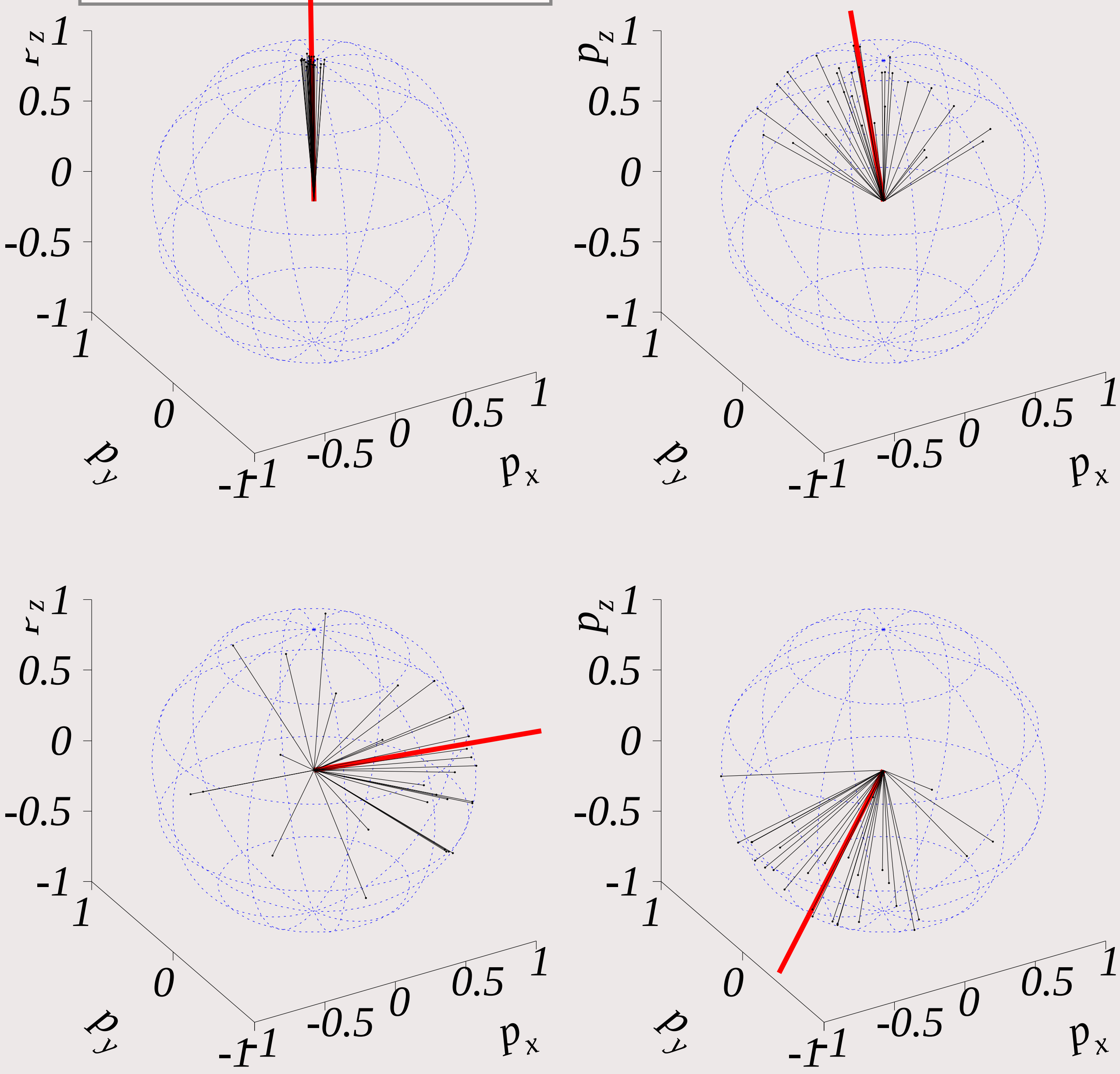


- NOT IDEAL due to synchr. osc. and SR noise effects: DECOHERENCE
- $P_z^f \neq P_{z\,FS}^f$
- $\epsilon(B_{RF})$, α

influence of spin decoherence on spin flip procedure efficiency???

1	idealized case: without synhr. osc.	$(\eta = 0)$	without SR	$(\alpha_s = 0,\, \kappa = 0)$
2	with synhr. osc.	$(\eta \neq 0)$	without SR	$(\alpha_s = 0,\, \kappa = 0)$
3	with synhr. osc.	$(\eta \neq 0)$	with SR with noise	$(\alpha_s \neq 0,\, \kappa \neq 0)$

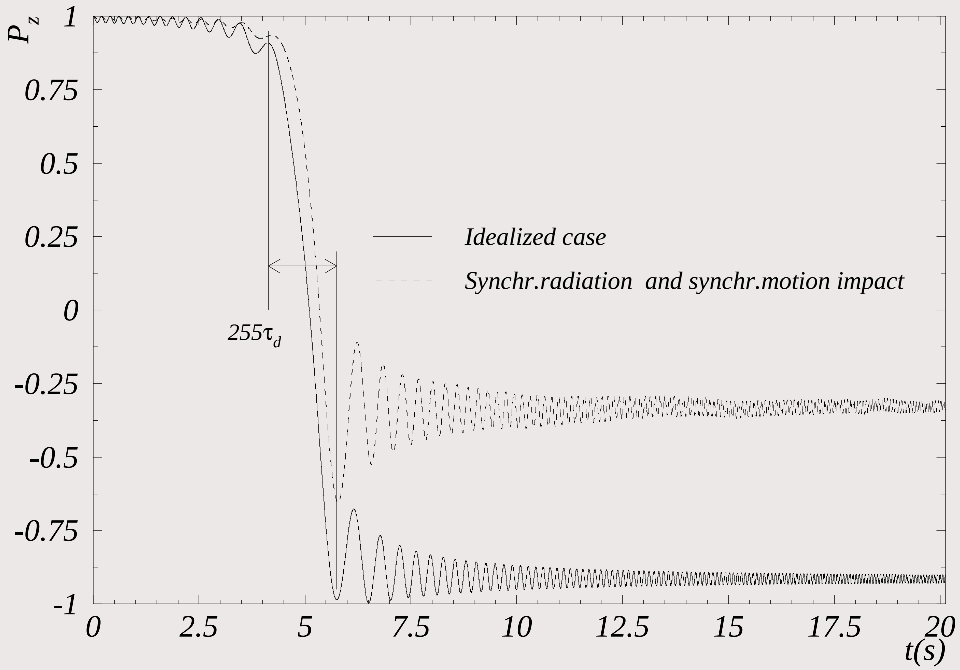
What is going on?



PREDICTION - Is spin flip on HERA possible?

- during flipping process spins pass through the horizontal plane
- if duration is large, spins might decohere due to synchr. osc. and synchrotron radiation
- indeed 2D modes (for HERA) predict: depolarization in horiz.plane

Buon 1987	total, τ_d=1 ms
Koutchouk 1991	total, τ_d=5.7 min
Heinemann 1997	diffusion→a limiting angle, τ_{d-h} = 6.66 ms

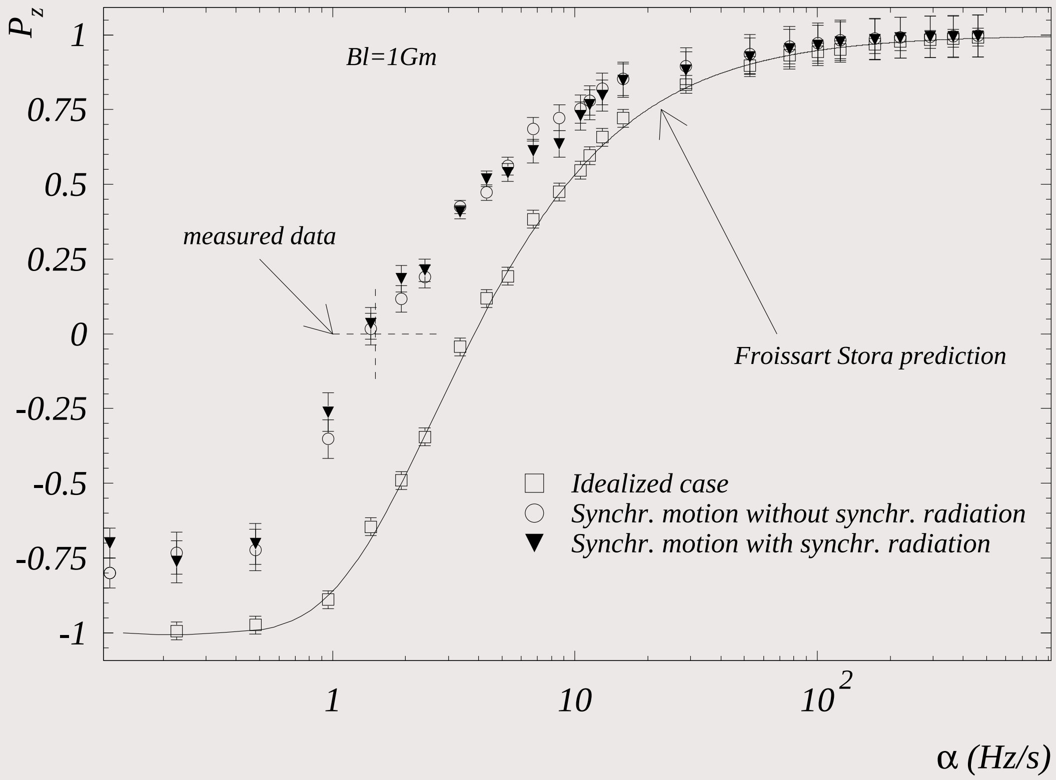


- a trial for HERA parameters: $B_{RF}l = 1Gm$, $\alpha = 0.9Hz/s$, ($1.5Hz/s$ decoherence)
- relatively high degree of polarization: $P_z^f \approx -0.3$ after spin reversal procedure, even with synchr.osc. and SR effects
- time the system spends in the resonance: 250 $\tau_{d-h} \gg$ 1ms - Buon decoherence time

AT LEAST partial polarization reversal is still possible also in high energy electron storage rings.

Polarization reversal ...

... as a function of α with constant RF field strength $\int B^0_{RF} dl =$ 1 Gm.



Classification with respect to time spent in resonance:

- short(high ramping rates) $\approx$ final P_z close to Froissart-Stora prediction
- longer (some $1/Q_s$)$\approx$ lower polarization reversal efficiency

- final P_z is lower due to synch.osc.
- SR influence is not strong

- measured point agrees with model prediction
  for parameters of HERA, optimal for depolarizing: $\alpha \approx$ 1.5 Hz/s, $\int B^0_{RF} dl = 1$ Gm
- optimal polarization reversal:
  $\alpha \approx$ 0.2 Hz/s (not tried at HERA)

How to explain the effect of synchrotron motion?

Spin precession in horiz.plane with significant/insignificant SR and without disturbing RF field

Spin prec.rate $\nu = a\gamma = \nu_0\,(1+\eta) = \nu_0(1+\eta_0\cos\psi_s)$, $\quad \psi_s = Q_s\theta + \psi_{s0}$, $\quad \theta = 2\pi s/L$

Therefore spin phase Ψ

ψ_{s0} is constant, but canNOT be neglected

- stochastics too weak - resonance crossings too short - to establish beam equil.!!!

$$\Psi = \int_0^\theta \nu d\theta' = \nu_0\theta + \nu_0\frac{\eta_0}{Q_s}\sin\psi_s \qquad -\nu_0\frac{\eta_0}{Q_s}\sin\psi_{s0}$$

and horizontal spin component - with expanding into series

$$\boxed{S_{horz}} \sim \boxed{e^{i\Psi}} = e^{i\nu_0\theta}e^{i\nu_0(\eta_0/Q_s)\sin\psi_s} \qquad \cdot e^{-i\nu_0(\eta_0/Q_s)\sin\psi_{s0}}$$

$$= e^{i\nu_0\theta}\sum_{m=-\infty}^{\infty} e^{im\psi_s}J_m\left(\frac{\eta_0\nu_0}{Q_s}\right) \qquad \cdot \sum_{m'=-\infty}^{\infty} e^{-im'\psi_{s0}}J'_m\left(\frac{\eta_0\nu_0}{Q_s}\right)$$

J_m is a Bessel function. After uniform - averaging over all synchrotron phases ψ_s:

$$\boxed{\langle e^{i\Psi}\rangle} = e^{i\nu_0\theta}\left\langle J_0\left(\frac{\eta_0\nu_0}{Q_s}\right)\right\rangle$$

Still we can - average over initial synchrotron phases ψ_{s_0}. We obtain:

$$\boxed{\langle e^{i\Psi}\rangle} = e^{i\nu_0\theta}\sum_{m=-\infty}^{\infty} e^{imQ_s\theta}J_m^2\left(\frac{\eta_0\nu_0}{Q_s}\right)$$

explicit dependence of θ, beating with frequency Q_s, i.e. recoherence every synhrotron periode

(when $e^{iQ_s\theta} = 1$)

We obtained:

$$\langle e^{i\Psi}\rangle = e^{i\nu_0\theta} \sum_{m=-\infty}^{\infty} e^{imQ_s\theta} J_m^2\left(\frac{\eta_0\nu_0}{Q_s}\right)$$

explicit dependency on θ, beating with frequency Q_s, i.e. recoherence every synchrotron period.

(when $e^{iQ_s\theta} = 1$)

We repeat (from the model with RF field):

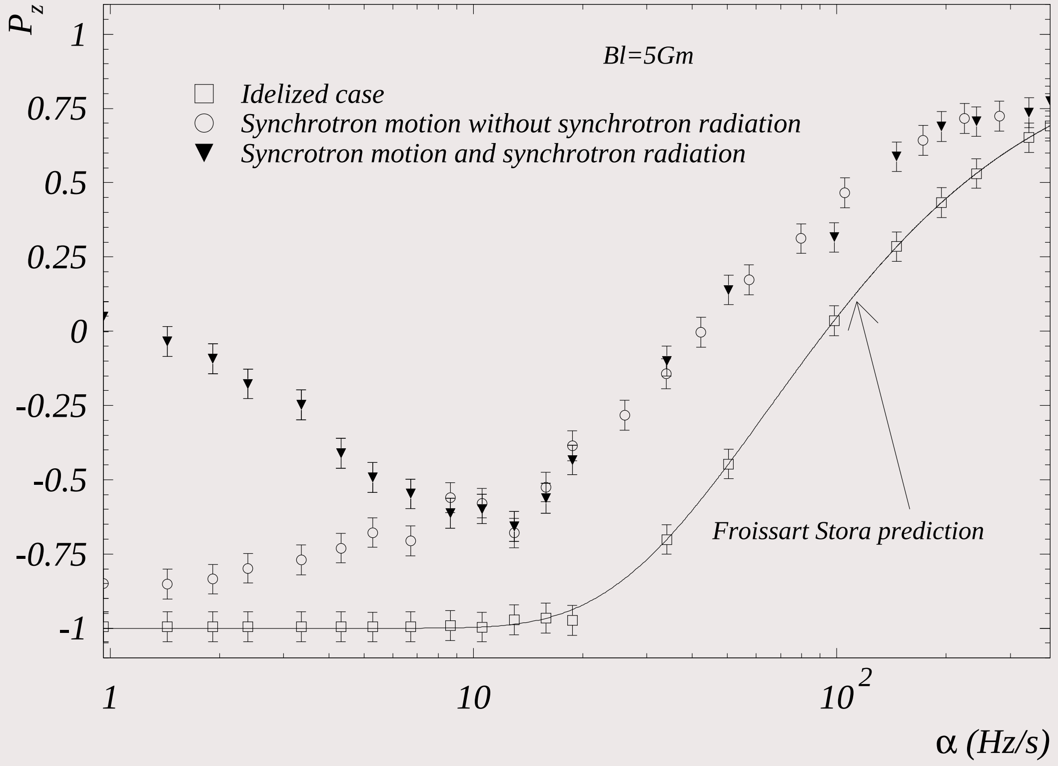
$$\frac{d\Psi}{ds} = -\Omega_r^0 \sin\Psi \frac{\cos\Theta}{\sin\Theta} + D(1+\eta) \qquad (2)$$

- travelling around the ring the spins spread out in Ψ due to synhr. motion (η recoherence)
- the term due to RF field introduces additional RF decoherence

complete inverse recoherence becomes impossible

Polarization reversal - with stronger RF fields

... as a function of α with constant RF field strength $\int B^0_{RF} dl =$ 5 Gm.



Classification with respect to time spent in resonance:

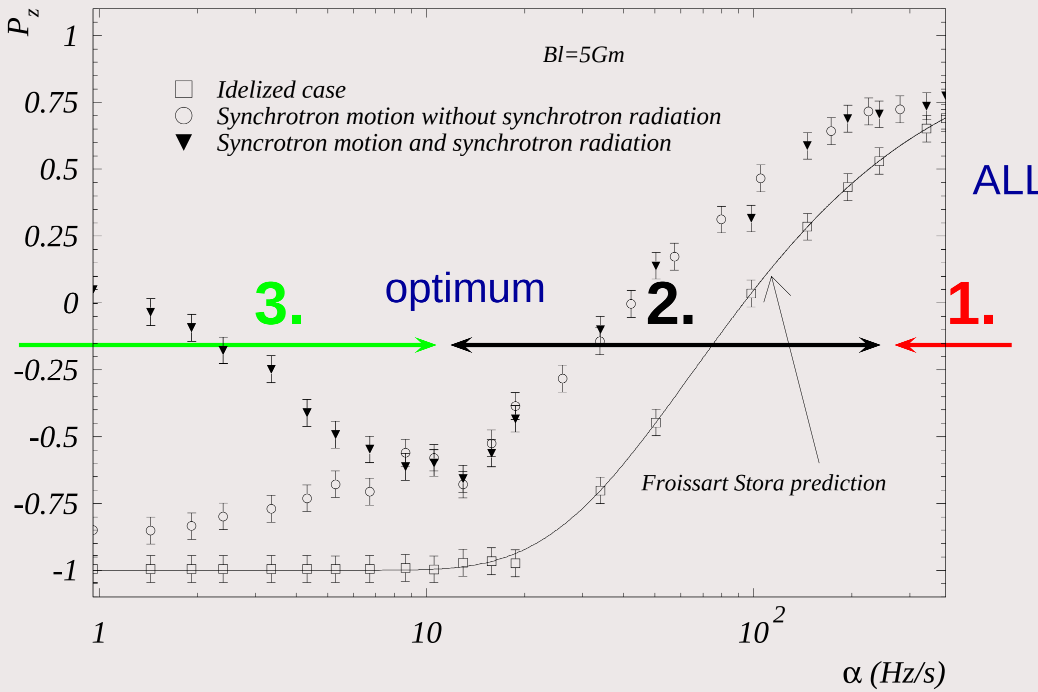
- short(high ramping rates) $\approx$ final P_z close to Froissart-Stora prediction
- longer (some $1/Q_s$)$\approx$ lower polarization reversal efficiency
- even longer (many $1/Q_s$, slow crossing of resonance) $\approx$
  - complete dopolarization at very low ramping rates α
  - strong SR influence

- optimal polarization reversal and optimal ramping rates for polarization reversal
- Sokolov-Ternov effect is negligible on this time scale

Not only that reverse recoherence becomes impossible,

stohastic influences of SR noise accumulate $\Rightarrow$ ACTIVE partial or total spin precession decoherence!

Three regions



ALL MODELS WERE RIGHT!

1. time in resonance region is short - non affected Froissart-Stora behaviour

2. time in resonance is of some $1/Q_s$, $\langle e^{i\Psi}\rangle = e^{i\nu_0\theta}\sum_{m=-\infty}^{\infty} e^{imQ_s\theta} J_m^2\left(\frac{\eta_0\nu_0}{Q_s}\right)$

- a shift to a lower polarization reversal efficiency due to RF field decoherence (on top of sychrotron motion recoherence beating)

3. synchr. motion completes many periodes during the time of spin reversal, orbital damping and noise exhibit equilibrium in the beam during this same time

- partial or complete depolarization due to SR effects decoherence (on top of synchr.motion and RF field decoherence)

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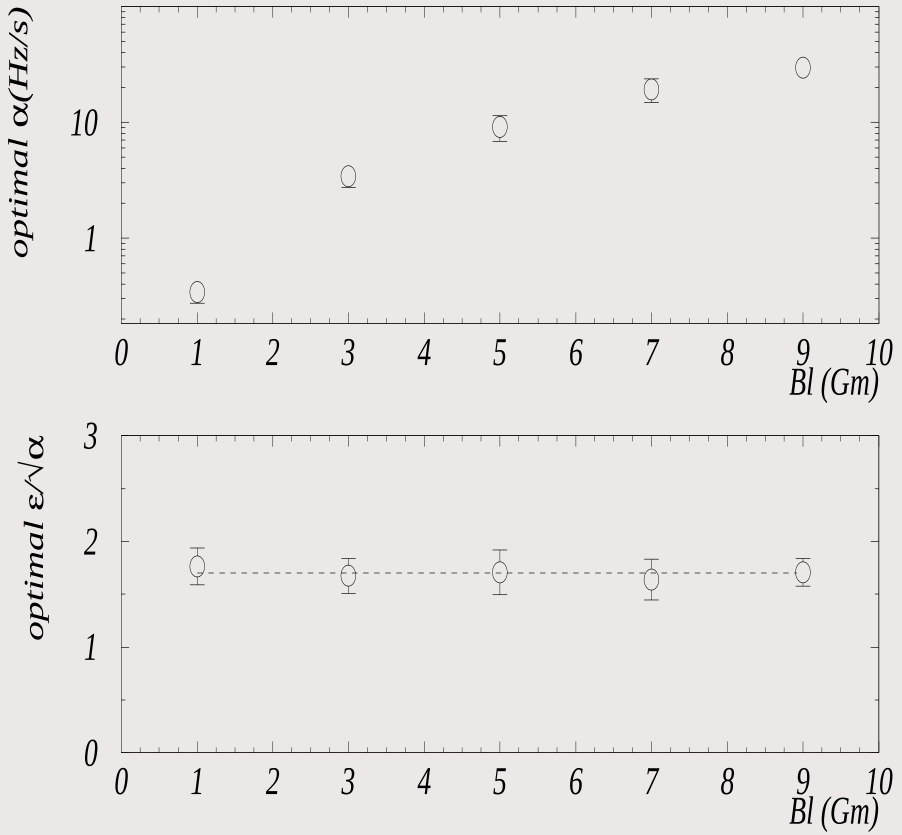
Dependence on RF field strength



- ramping rate α and dimensionless parameter $\epsilon/\sqrt{\alpha}$ (of $\int B^0_{RF} dl$) optimal for polarization reversal in HERA
- $\epsilon/\sqrt{\alpha}$ is INDEPENDENT of $\int B^0_{RF} dl$ $\rightarrow$ (with stronger RF field the resonance can be crossed faster with same final effect)
- for HERA optimal $\epsilon/\sqrt{\alpha}$ is 1.7±0.2 and minimal *final* vertical component of polarization P_z is around -0.75

no sextupole effects, no spin rotators

CONCLUSION

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MODEL

- Model describes spin motion in 3D, includes particle synchrotron oscillations, SR (synchrotron radiation) influences and RF radial disturbing magnetic field kicker.

FINDINGS

- Even for a high energy storage ring HERA with significant synchrotron radiation and synchrotron motion, polarization reversal is possible with relatively small strengths of radial RF magnetic field.
- Due to RF field and SR decoherence (on top of sychrotron motion recoherence beating), the spin angles mix up during the passage through the horizontal plane, which results in effectively lower polarization reversal efficiency:
  - if synchrotron motion completes some periodes in the resonance, the influence of SR noise (on polarization reversal) is small,
  - however, with slow enough crossing, effects of SR noise significantly lower the degree of polarization reversal.

Although in reality additional fields affect the spin motion, the presented model serves as
a good basis to distinguish different effects influencing
the spin motion of high energy electrons.

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THANKS TO

- Institute DESY, Hamburg, Germany
- Institute Jožef Stefan, Ljubljana, Slovenija
- Desmond Barber

DEDICATED TO

my children *Svit, Luna, Dora*

STUDY OF POSSIBILITIES FOR A SPIN FLIP IN HIGH ENERGY ELECTRON STORAGE RING HERA

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Institute Jožef Stefan, University of Ljubljana

Kyoto, 2.10.2006
SPIN 2006

- Introduction
- Motivation
- Spin tune and decoherence of particle spins
- Model
  - Differential equations of spin motion
  - Differential equations of orbital motion
- Implementation (Fokker-Planck : Monte Carlo?)
- Model predictions: Spin/Polarization reversal - is it possible, to what extent, conditions and why?
- Summary