A Finite Element Model for the Thermal Transport in Solid Targets

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POLARIZED TARGET BONN

INS-BONN

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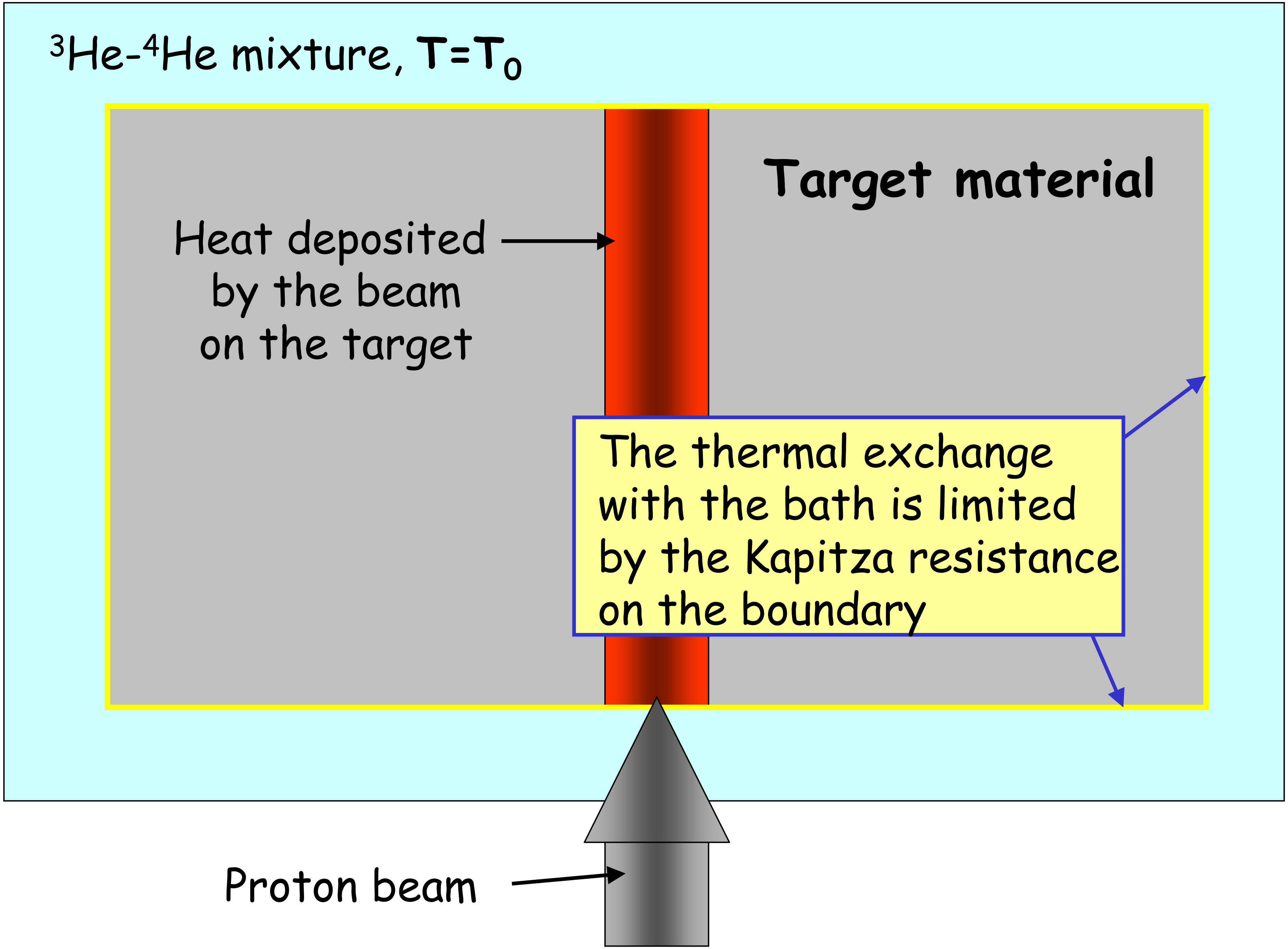
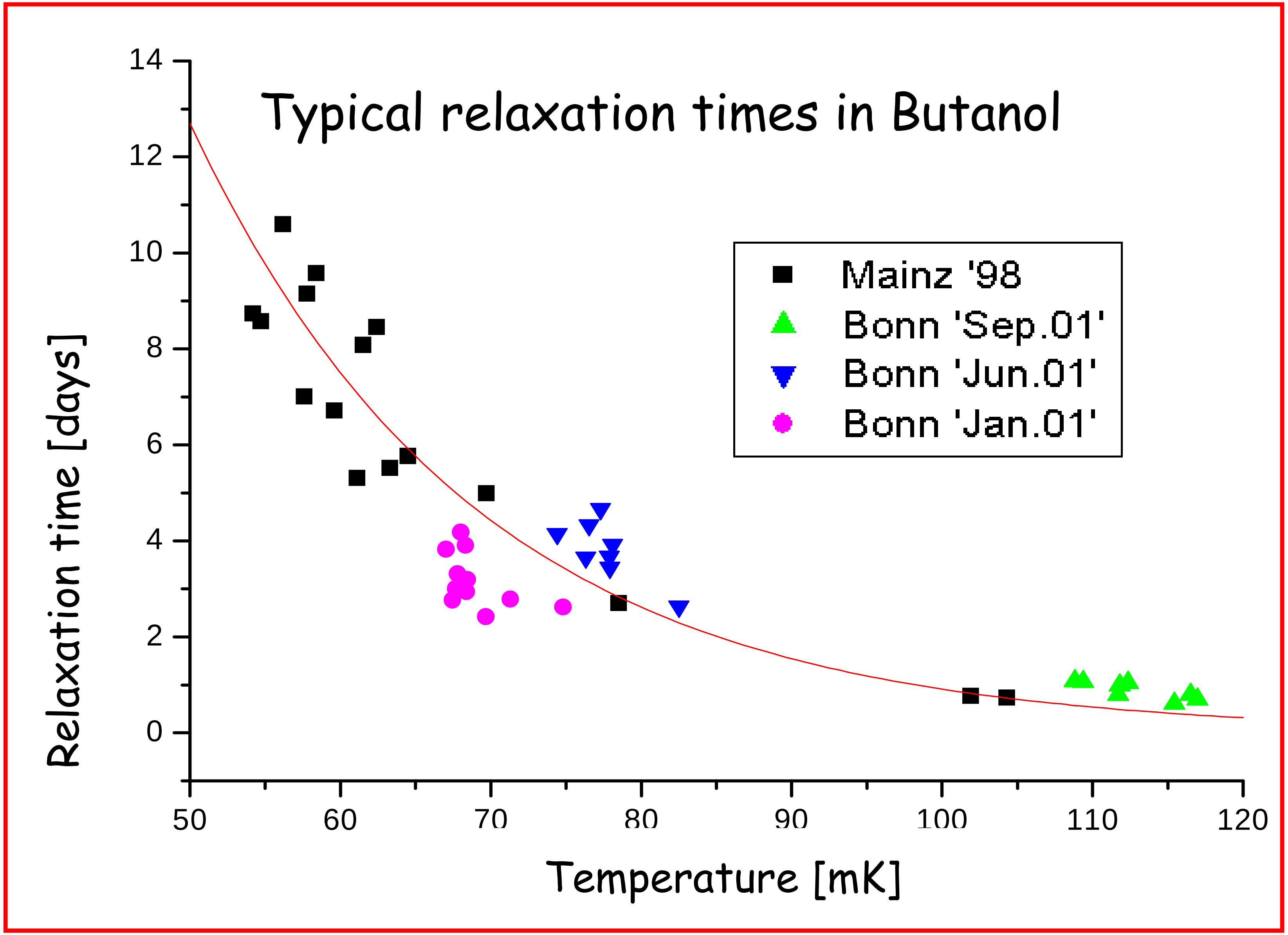

^{3}He-^{4}He mixture, **T=T$_0$**
Target material
Heat deposited
by the beam
on the target
The thermal exchange
with the bath is limited
by the Kapitza resistance
on the boundary
Proton beam

Temperature affects polarization

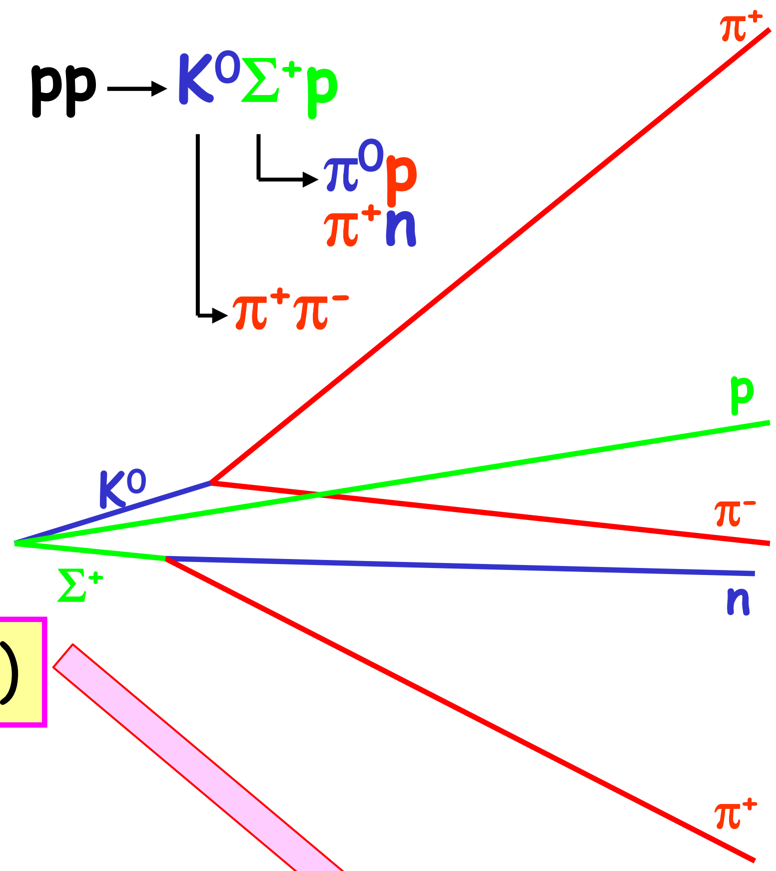


$$pp \rightarrow \Sigma^+\Theta^+ \rightarrow \Sigma^+K^0p$$

$pp \rightarrow K^0\Sigma^+p$

$\Sigma^+ \rightarrow \pi^0 p$, $\pi^+ n$

$K^0 \rightarrow \pi^+\pi^-$



Requirements:

★ For detection statistics:

high beam intensity ($N=10^7$/s)

★ For accuracy of vertex reconstruction:

high beam collimation (~$1mm^2$)

detection of Σ^+ ($c\tau \approx 2.4$cm)

Target design

Frozen spin mode

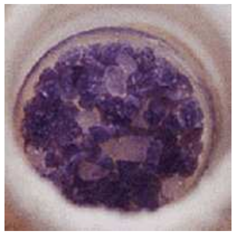
The Kapitza resistance

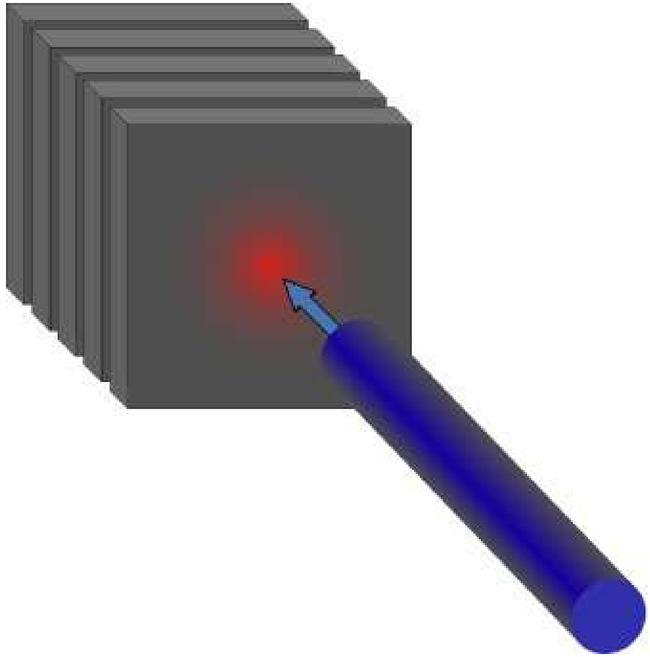
From the acoustic mismatch theory

$$\frac{\dot{Q}}{A} = \frac{\pi^2 k_B^4 T^4 \rho_h v_h}{30\hbar \rho_s v_s^3}$$

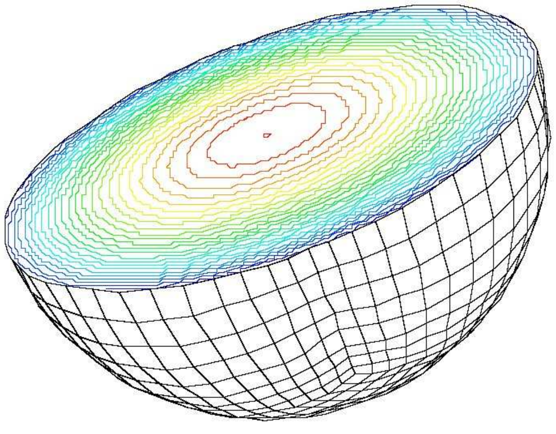
$$\frac{\dot{Q}}{A} = \alpha\left(T^4 - T_0^4\right)$$

Heat exchanged between the solid and the bath at $T=T_0$





An analytical solution exists only for a sphere with thermal conductivity k=const.



$$-k\nabla^2 T = \dot{Q}_V \qquad (k = \text{const.})$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)T(r) + \frac{\dot{Q}_V}{k} = 0$$

$$T(r) = -\frac{\dot{Q}_V}{6k}\cdot r^2 + const.$$

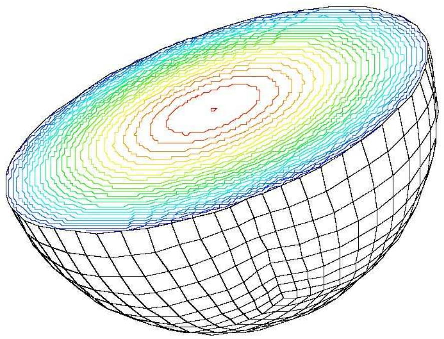
$$\Delta T = T(0) - T(R) = \frac{\dot{Q}_V}{6k}\cdot R^2$$

On the surface:

$$\frac{\dot{Q}}{A} = \alpha\left(T^4 - T_0^4\right)$$

The sphere is the „worste case"

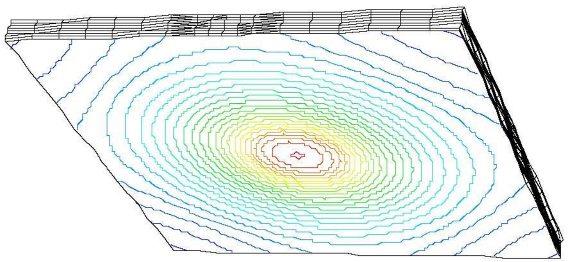
$$\frac{\dot{Q}}{A} = \alpha\left(T^4 - T_0^4\right)$$



$$T = \sqrt[4]{\frac{\dot{Q}}{A \cdot \alpha} + T_0^4}$$

$$T = \sqrt[4]{\frac{\dot{Q}_v \cdot \frac{4}{3}\pi R^3}{\alpha \cdot 4\pi R^2} + T_0^4}$$

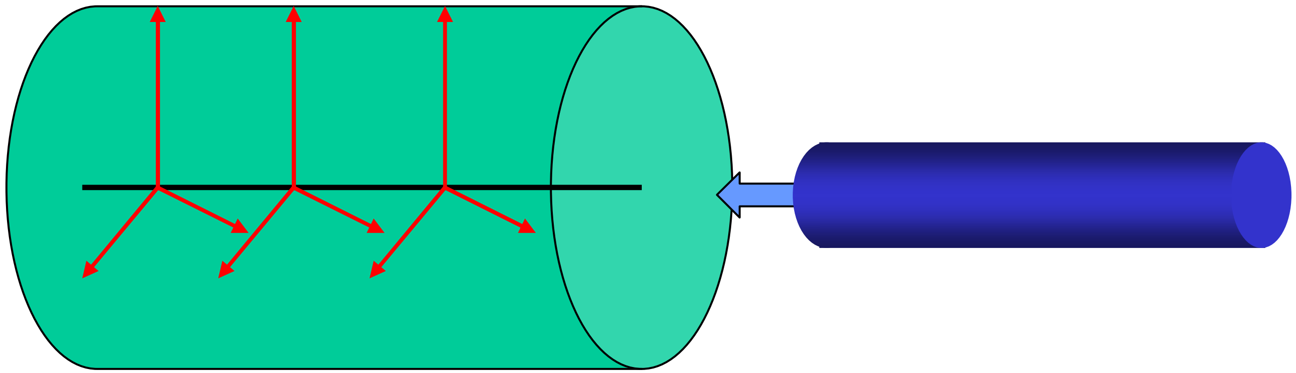
$$T = \sqrt[4]{\frac{\dot{Q}_v}{3\alpha} R + T_0^4}$$



Thickness decreases, area ≈ constant

Other common assumptions:

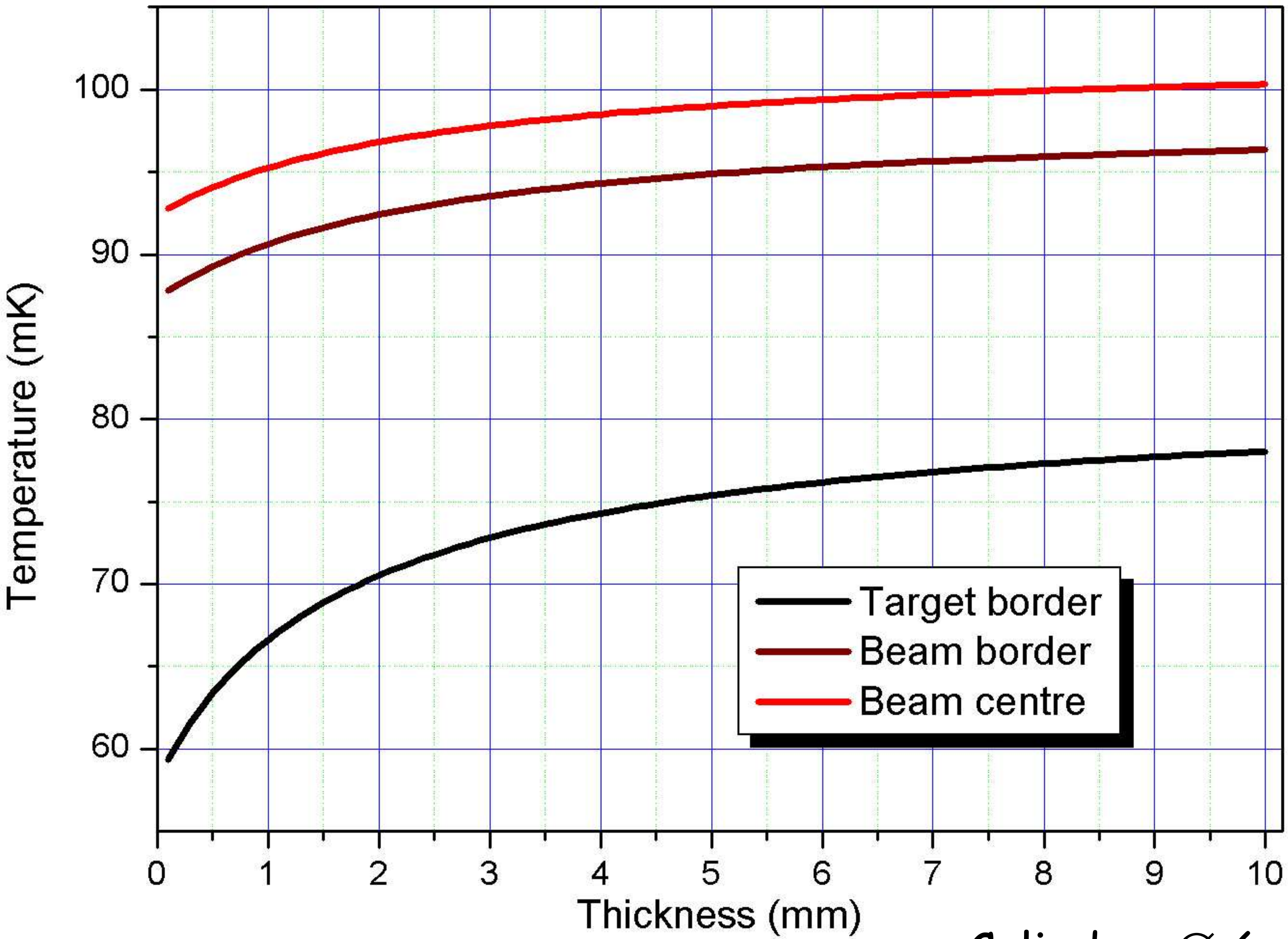
1) The heat is dissipated through the lateral surface only



2) The Kapitza "jump" is constant on the surface:

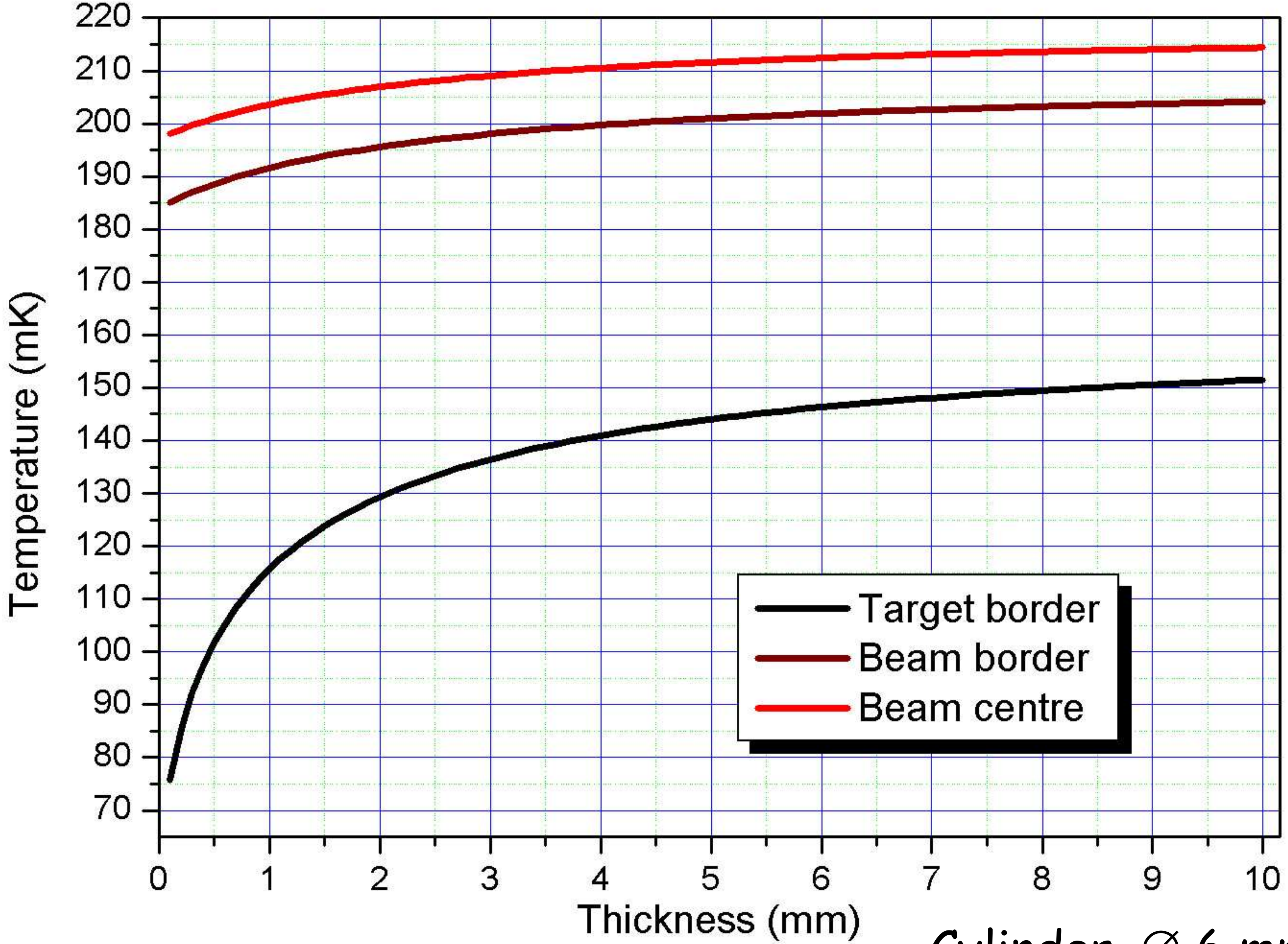
$$\frac{\dot{Q}}{A} = \alpha\left(T^4 - T_0^4\right) \qquad T = \sqrt[4]{\frac{\dot{Q}}{A \cdot \alpha} + T_0^4}$$

N = 500 kHz, beam radius = 0.56 mm



Cylinder, ∅ 6 mm

N = 10MHz, beam radius = 0.56 mm



Cylinder, ∅ 6 mm

The model - (1) Heat-conduction equation

The time-independent heat-conduction equation

$$-k\nabla^2 T = -k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) = q$$

(Poisson eq.)

for a non constant thermal conductivity, *k*, is:

$$-\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) = q$$

$$k = k(T) = aT^{\beta}$$

with a and β free parameters

The model - (2) The Kapitza resistance

$$\dot{Q} = -kA\frac{\partial T}{\partial n}$$

+

Robin boundary condition

$$\frac{\partial T}{\partial n} + h \cdot T = f$$

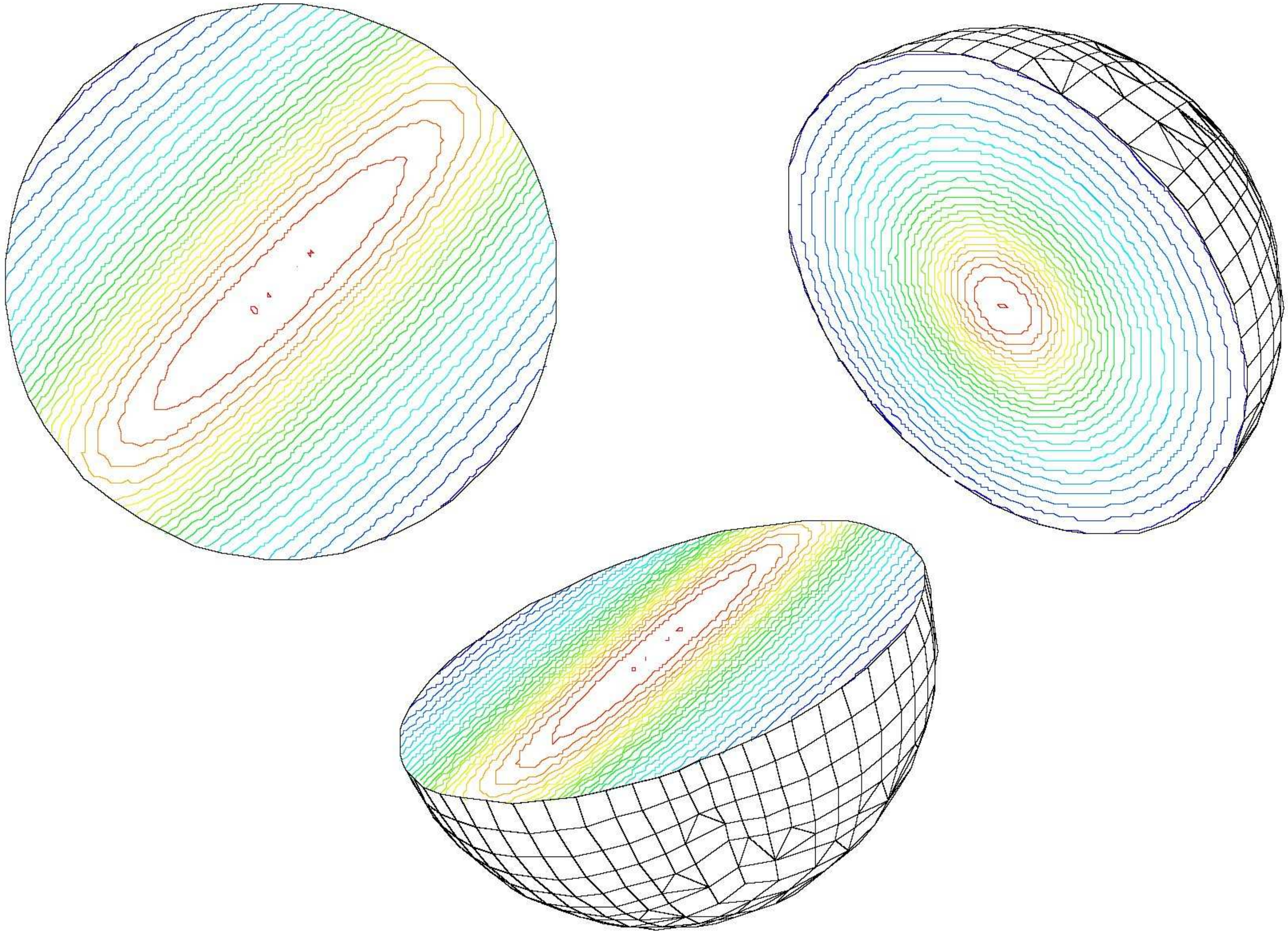
$$\frac{\dot{Q}}{A} = \alpha\left(T^4 - T_0^4\right)$$

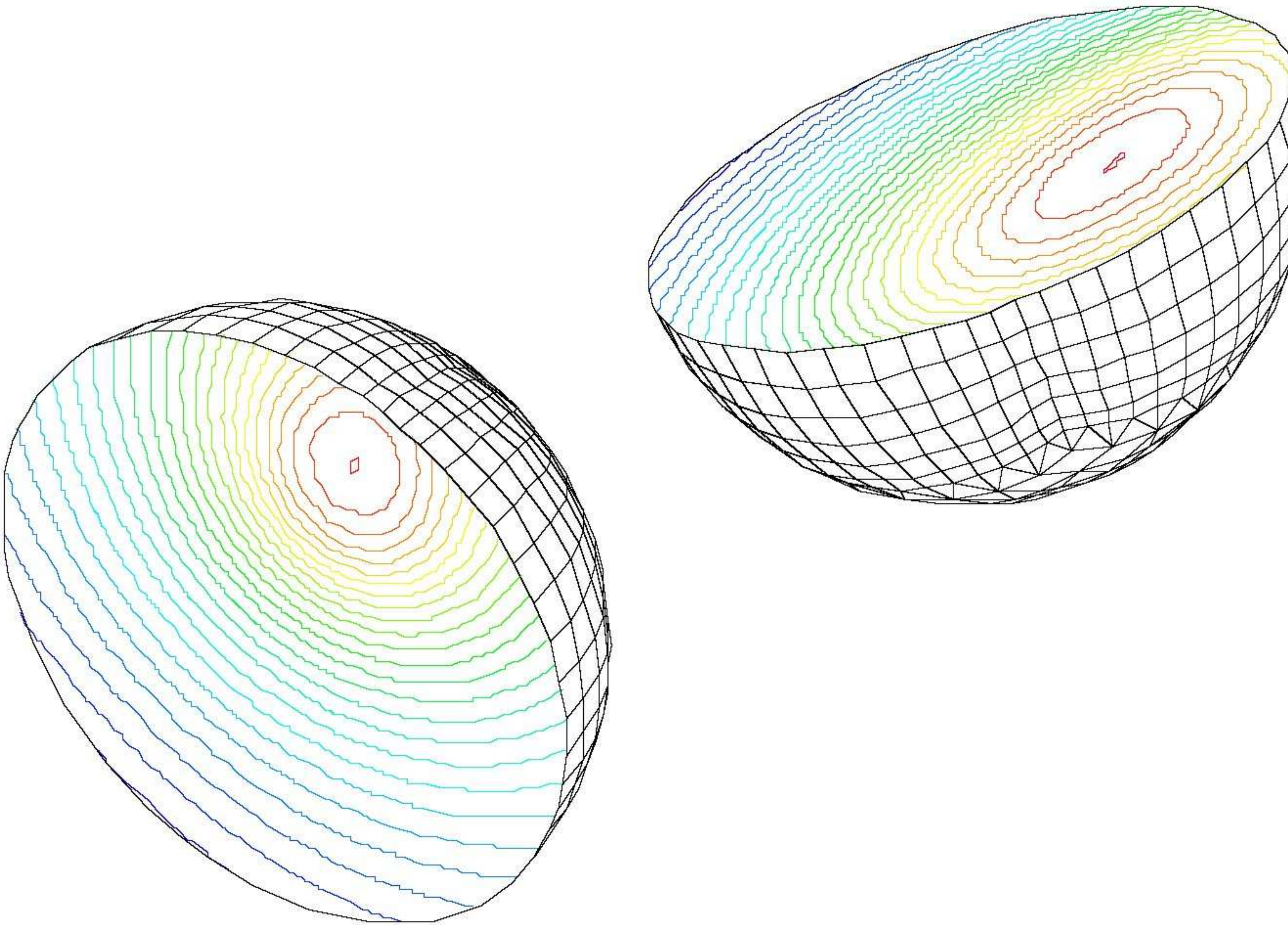
$$\Rightarrow$$

$$k\frac{\partial T}{\partial n} + \alpha \cdot T^4 = \alpha \cdot T_0^4$$

The implementation of the model is based on the UG package. *P. Bastian et al., Comput. visual. Sci. 1(1997)*

The solution of the linear system is based on the multigrid method. + adaptive grids





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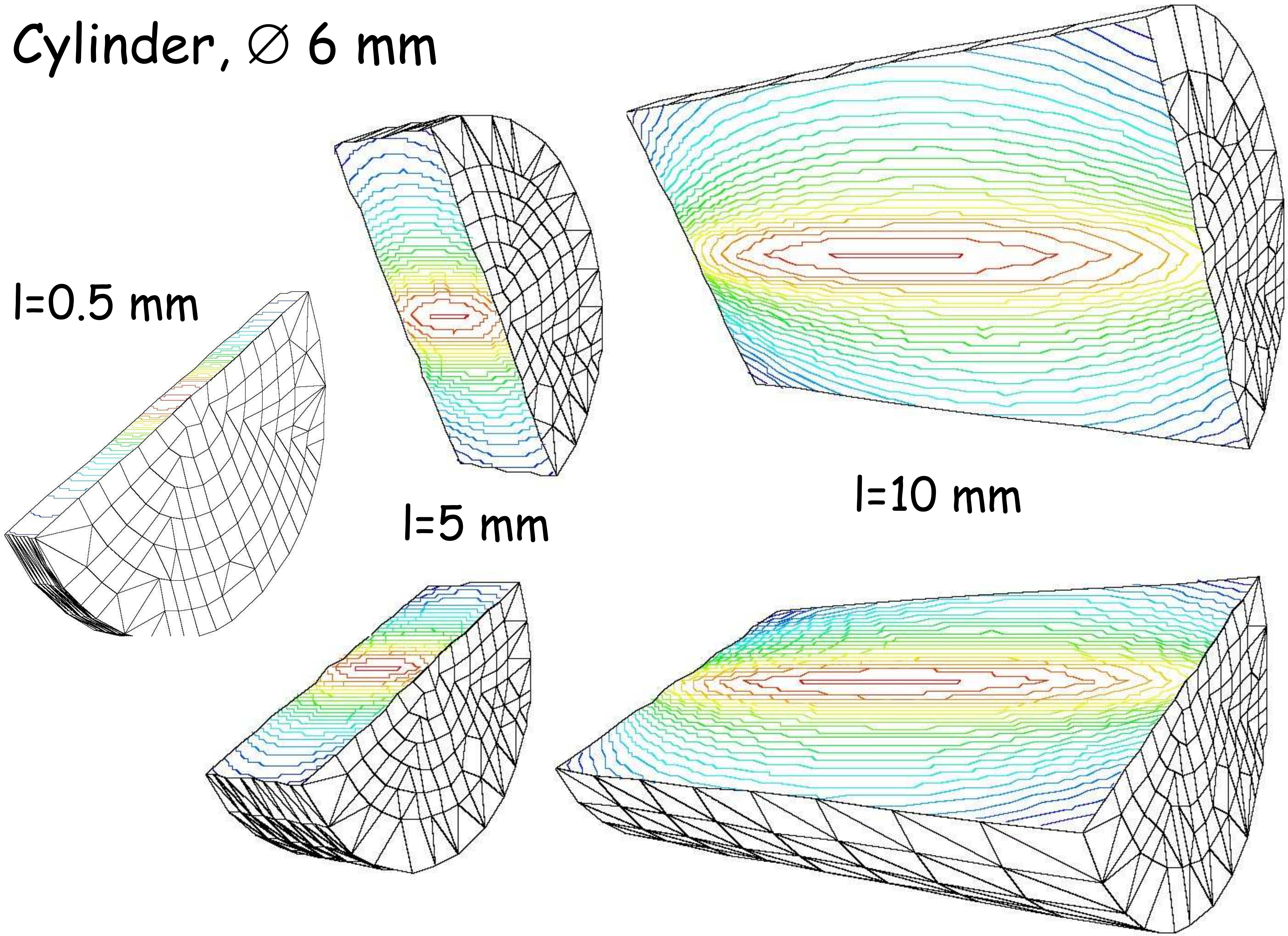

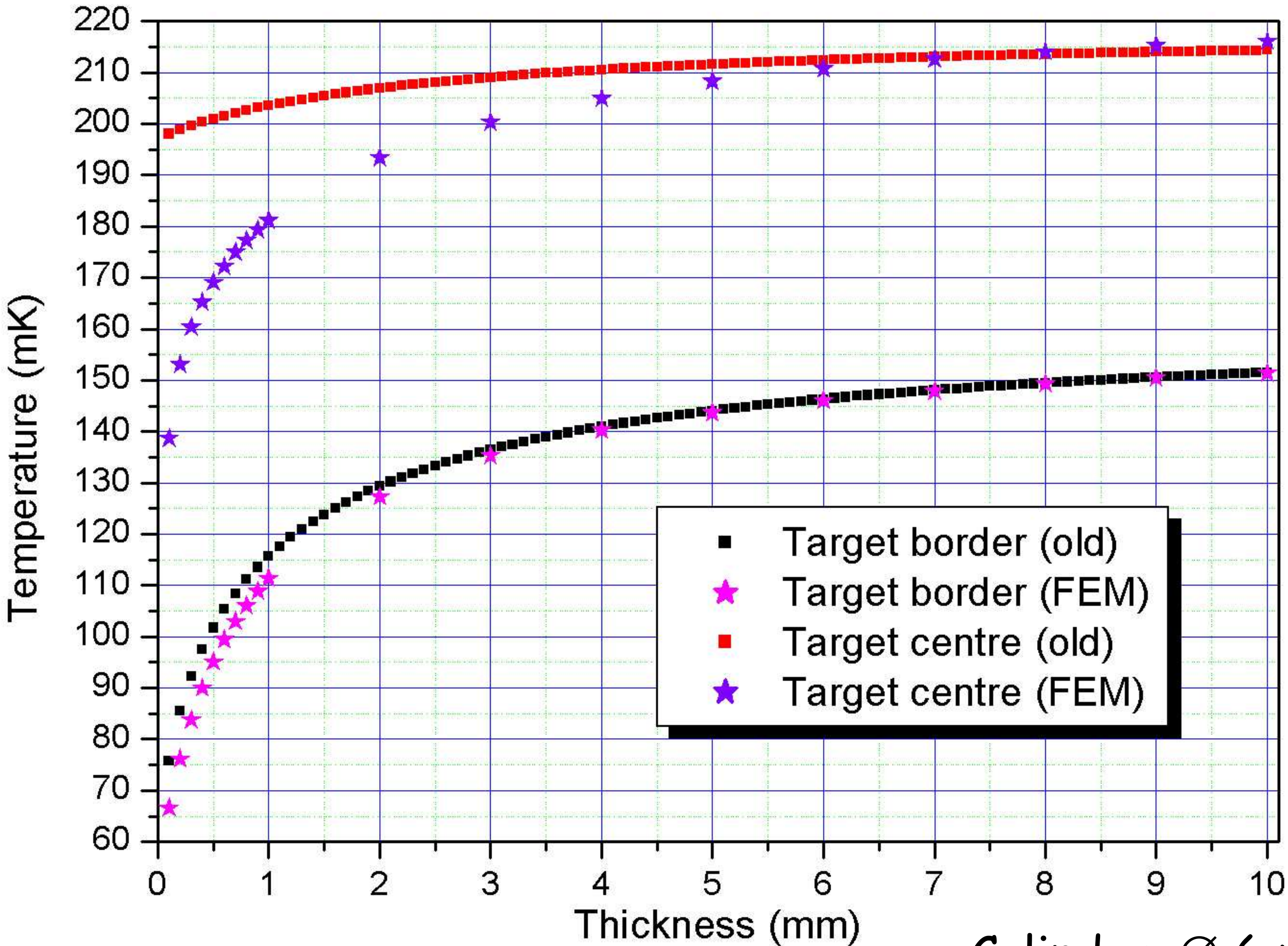

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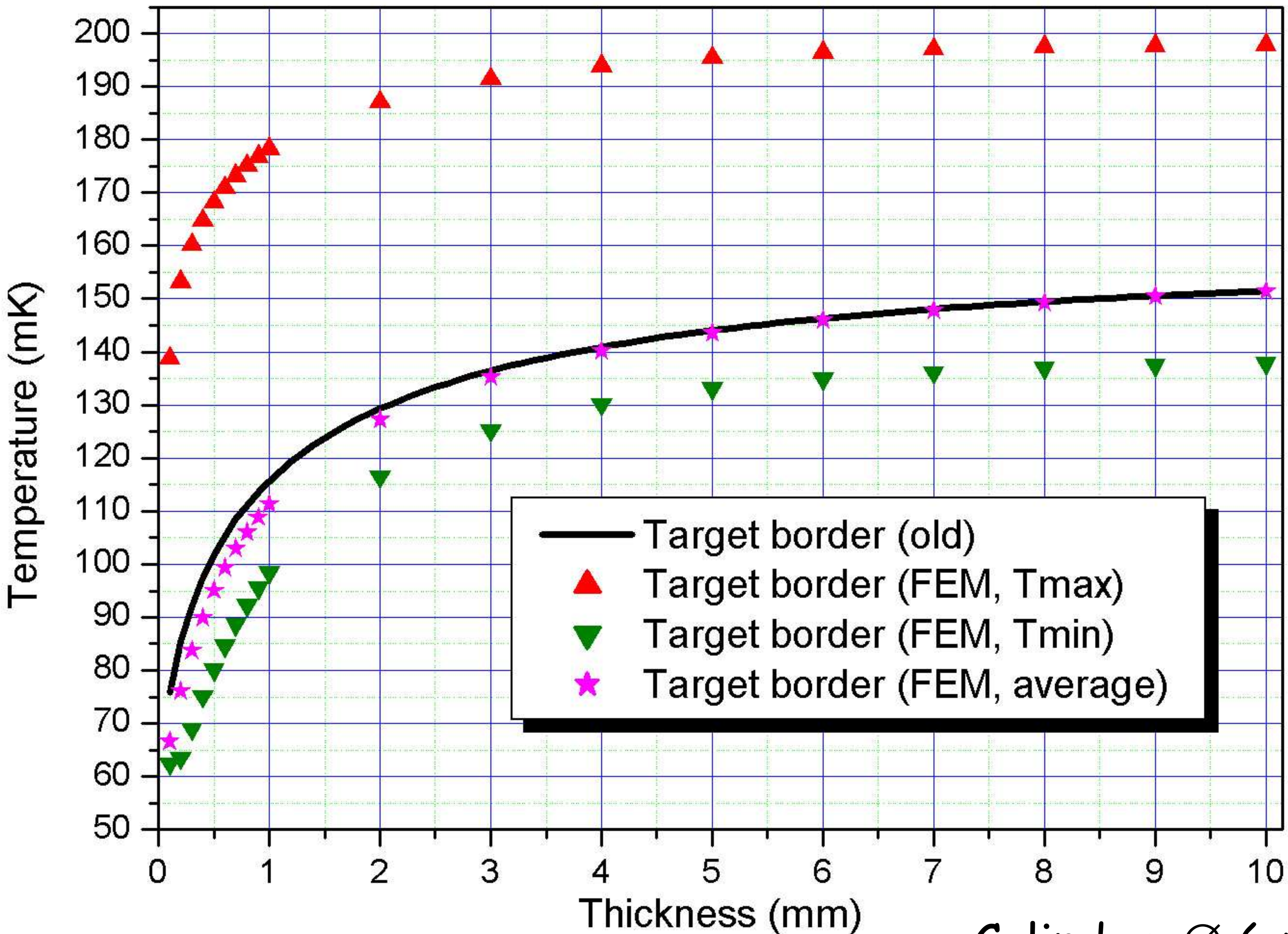

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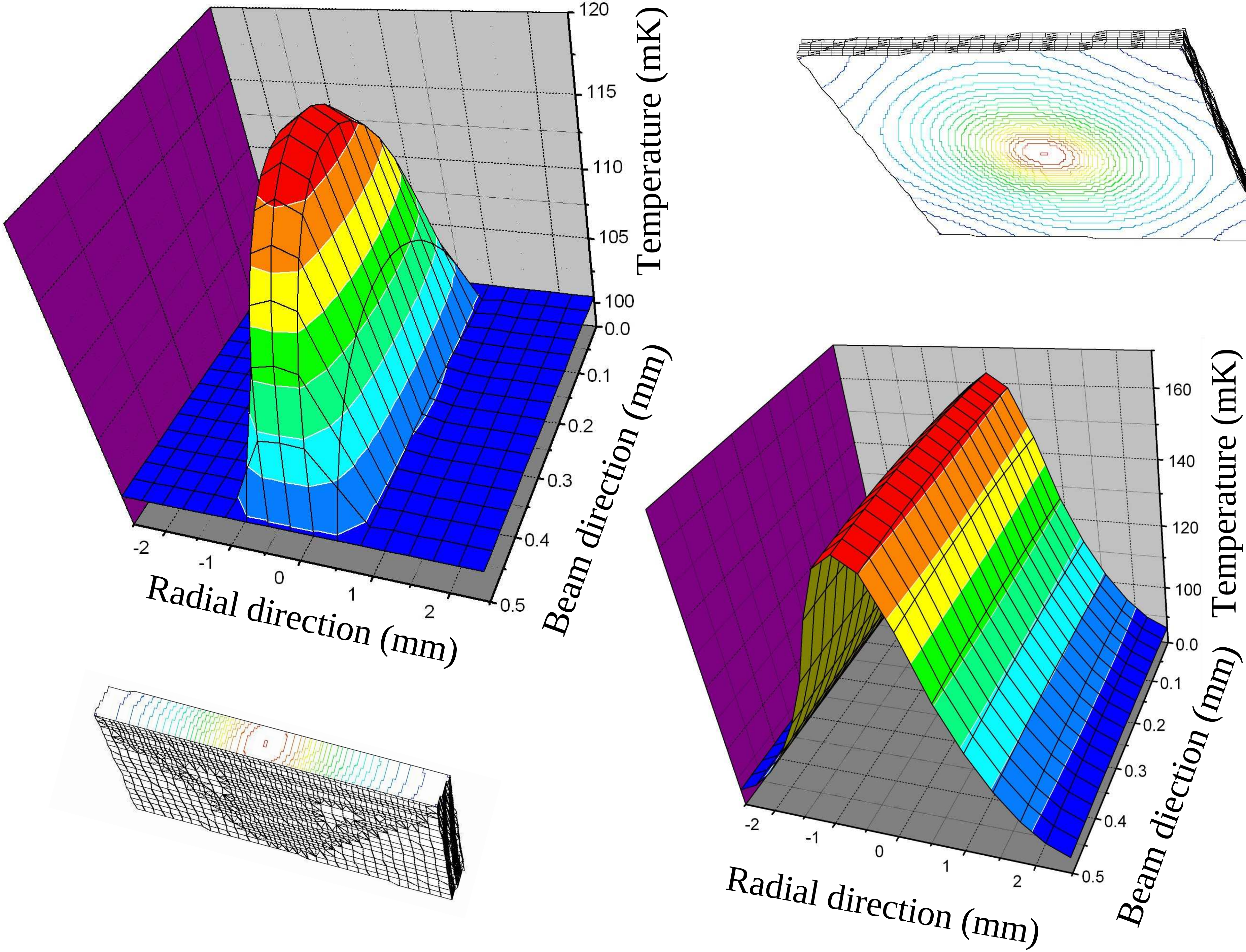
120
115
110
105
100
0.0
0.1
0.2
0.3
0.4
0.5
Temperature (mK)
Beam direction (mm)
-2
-1
0
1
2
Radial direction (mm)
160
140
120
100
0.0
0.1
0.2
0.3
0.4
0.5
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Beam diection (mm)
-2
-1
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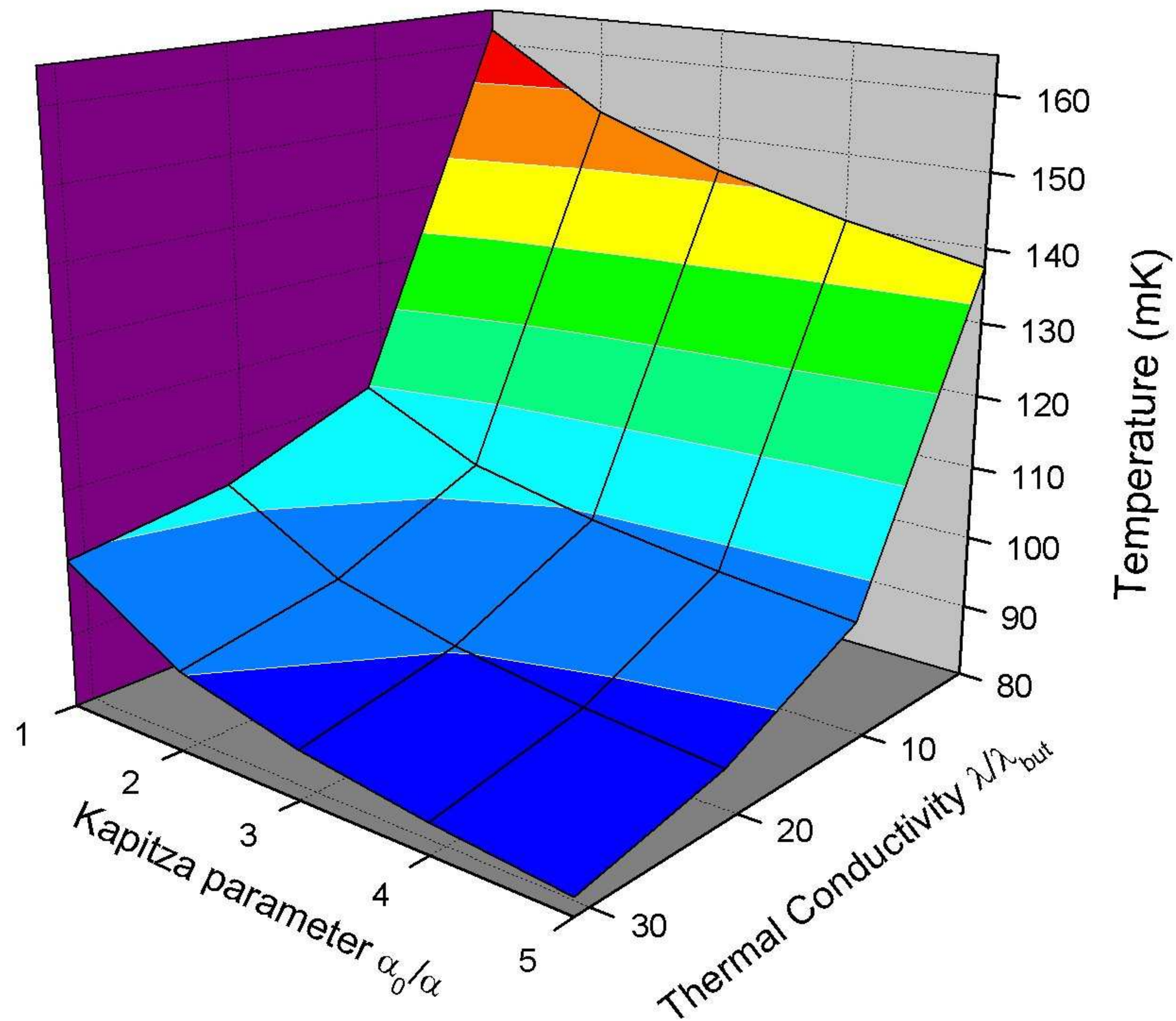
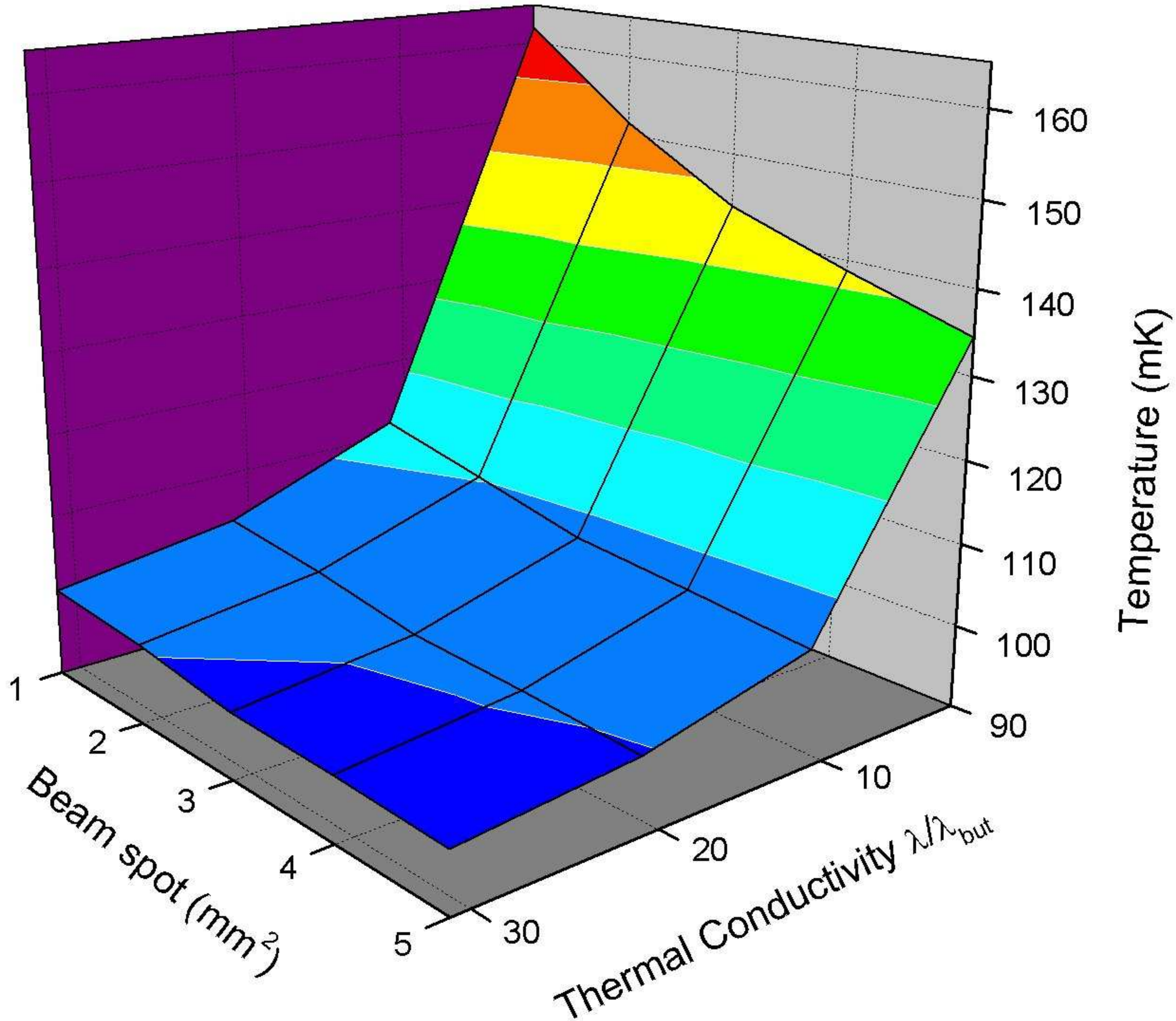
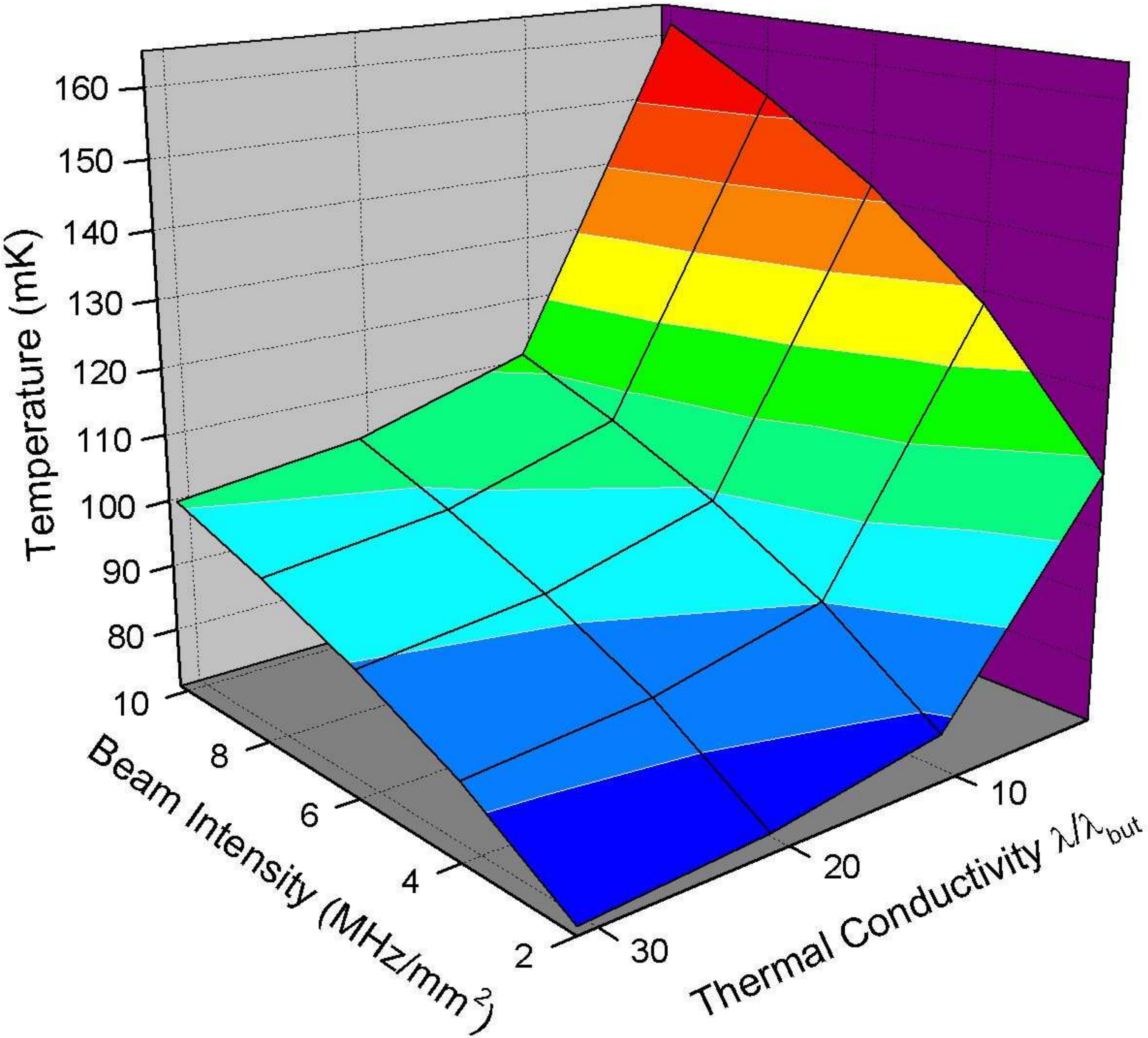


160
150
140
130
120
110
100
90
80
Temperature (mK)
1
2
3
4
5
Kapitza parameter α_0/α
10
20
30
Thermal Conductivity λ/λ_{but}



160
150
140
130
120
110
100
90
Temperature (mK)
1
2
3
4
5
Beam spot (mm^2)
10
20
30
Thermal Conductivity λ/λ_{but}



Temperature (mK)
160
150
140
130
120
110
100
90
80
10
8
6
4
2
Beam Intensity (MHz/mm^2)
10
20
30
Thermal Conductivity λ/λ_{but}

CONCLUSIONS

What do we learn?

- Reducing the thickness is not the ultimate solution

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THANK YOU!