THE QCD ANALYSIS OF THE WORLD DATA ON STRUCTURE FUNCTIONS $g_1^{p,d,n}$ FOR PROTON, DEUTERIUM AND NEUTRON

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On behalf of the COMPASS collaboration

LIST OF DATA

-List of data sets used in the present analysis

Exp.	Target nucleon	Nr. of points	Reference
EMC	p	10	Nucl. Phys. B 328 (1989) 1
SMC	p	12	Phys.Rev. D 58 (1998) 112001
SMC	d	12	id.
COMPASS	d	43	hep-ex/0609038, submitted to PLB
E143	p	28	Phys.Rev. D 58 (1998) 112003
E143	d	28	id.
E155	d	24	Phys. Lett. D 463 (1999) 339
E155	p	24	Phys.Lett. B 493 (2000) 19
JLAB	n	3	Phys. Rev. Lett. 92 (2004) 012004
E142	n	8	Phys.Rev. D 54 (1996) 6620
E154	n	11	Phys.Rev. Lett. 79 (1997) 26
HERMES	n	9	Phys.Lett. B 404 (1997) 383
HERMES	p	9	Phys.Rev. D75 (2005) 012003
HERMES	d	9	id.
Total		230	

See Catarina's Quintans talk

-Input for analysis: $g_1^p(x,Q^2), g_1^n(x,Q^2), g_1^N = \frac{1}{2}(g_1^p + g_1^n) = \frac{g_1^d}{1-1.5\omega_D}$

- usual cut $Q^2 > 1$ GeV^2 limits the x range, for COMPASS data $x > 0.004$

-two additional points form COMPASS at $Q^2 > 0.7$ GeV^2: $x = 0.0030 – 0.0035$ and $x = 0.0035$-0.0040 not used in QCD fits

g_1 @ NLO

In QPM g_1 is related to the polarized parton distribution functions (PDF):

$$g_1^{p(n)}(x,Q^2) = \frac{1}{9}\left(C_{NS} \otimes \left[\pm\frac{3}{4}\Delta q_3 + \frac{1}{4}\Delta q_8\right] + C_S \otimes \Delta\Sigma + C_G \otimes \Delta G\right)$$

Where C_{NS}, C_S and C_G are Wilson coefficients,

Δq_3, Δq_8 - non-singlet polarized quark DF,

$\Delta\Sigma$ - singlet polarized quark DF,

ΔG - polarized gluon DF,

$\otimes$ - convolution: $a(x) \otimes b(x) = \int_x^1 \frac{dy}{y} a\left(\frac{x}{y}\right) \cdot b(y)$.

In the 3 quark limits:

$\Delta\Sigma = \Delta u + \Delta d + \Delta s$,

$\Delta q_3 = \Delta u - \Delta d$,

$\Delta q_8 = \Delta u + \Delta d - 2\Delta s$

FITTING PROGRAMS

PROGRAM 1 **[SMC, P.R. D58 (1998) 112002]**

numerical solutions of the DGLAP evolution equations for PDF's.

PROGRAM 2 **[Refered to in P.R. D70 (2004) 074032].**

Works in two steps:

1. **Analytical solution of the evolutions equations for the PDF moments,**
2. **Inverse Mellin transformation of moments for PDF's reconstruction**
   (similar to one developed for the QCD analysis of F_2 (x, Q^2), [Krivokhizhin et al., Z.Phys. C36 (1987) 51])

Both programs work in the $\overline{MS}$ renormalization and factorization scheme in next-to-leading (NLO) approximation and require input parametrizations of PDF's

DGLAP EVOLUTION EQUATIONS

$$\frac{d}{dt}\Delta q_{NS} = \frac{\alpha_s(t)}{2\pi} P_{qq}^{NS} \otimes \Delta q_{NS}$$ (non – singlet),

$$\frac{d}{dt}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{pmatrix} P_{qq}^S & 2n_f P_{qG}^S \\ P_{Gq}^S & P_{GG}^S \end{pmatrix} \otimes \begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix}$$ (singlet & gluon),

where $t = \log\left(Q^2/\Lambda^2\right)$ and P_{qq}, P_{qG}, P_{Gq} are polarized splitting functions.

EVOLUTION OF MOMENTS

1. $$\frac{d}{dt}\Delta q_{3(8)}^{(n)}(Q^2) = \frac{\alpha_s(t)}{2\pi}\gamma_{NS}\Delta q_{3(8)}^{(n)}(Q^2)$$ *(non-singlet sector),*

$$\frac{d}{dt}\begin{pmatrix}\Delta\Sigma^{(n)}\left(Q^2\right)\\ \Delta G^{(n)}\left(Q^2\right)\end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{pmatrix}\gamma_{qq} & \gamma_{qg}\\ \gamma_{gq} & \gamma_{gg}\end{pmatrix}\times\begin{pmatrix}\Delta\Sigma^{(n)}\left(Q^2\right)\\ \Delta G^{(n)}\left(Q^2\right)\end{pmatrix}$$ (singlet & gluon sector),

where $$\Delta q^{(n)}\left(Q^2\right) = \int_0^1 dx x^n \Delta q\left(x, Q^2\right),$$

γ_{ij} - anomalous dimensions.

2. $$\Delta q\left(x, Q^2\right) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dn x^{-n}\Delta q^{(n)}$$

INPUT PARAMETRIZATIONS

-The PDF $\Delta\Sigma$, Δq_3, Δq_8 and ΔG at $Q_0^2 = 3\ GeV^2$ are parametrized as:

$$\Delta F_k(x) = \eta_k \frac{x^{\alpha_k}(1-x)^{\beta_k}(1+\gamma_k x)}{\int_0^1 x^{\alpha_k}(1-x)^{\beta_k}(1+\gamma_k x)\,dx}, \qquad \eta_k = \int \Delta F_k(x)\,dx$$

-η_3, η_8 are fixed by the barion octet constants F&D assuming $SU(3)_f$ flavor symmetry:

$\eta_3 = F+D$, $\eta_8 = 3F-D$.

-The linear term $\gamma_k x$ used for $\Delta\Sigma$ only.

-Positivity limits $|\Delta s(x)| \leq s(x)$ & $|\Delta G(x)| \leq G(x)$ imposed at each step.

-Unpolarized PDF's are taken from MRST parametrizations

(Martin et al., Eur.Phys. J.C4(1998) 463).

- Finally, there are 10 free parameters determined by minimizations of the sum (MINUIT):

$$\chi^2 = \sum_{i=1}^{230} \frac{\left[g_1^{fit}\left(x_i, Q_i^2\right) - g_1^{\exp}\left(x_i, Q_i^2\right)\right]^2}{\left[\sigma\left(x_i, Q_i^2\right)\right]^2}.$$

FITTED PDF PARAMETERS

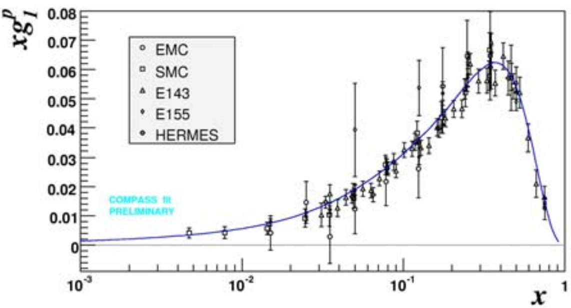
Both programs give consistent values of fitted PDF parameters with similar χ^2 for two solutions, one with $\Delta G > 0$, the other with $\Delta G < 0$:

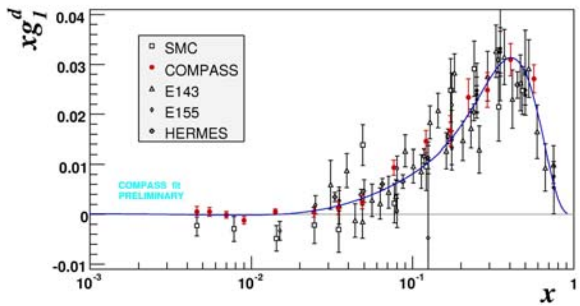
$\Delta G > 0$		
	Prog. Ref. [28]	Prog. Ref. [29]
η_Σ	0.276 ± 0.013	0.288 ± 0.011
α_Σ	$-0.285^{+0.073}_{-0.085}$	$-0.187^{+0.072}_{-0.065}$
β_Σ	$3.61^{+0.26}_{-0.24}$	$3.81^{+0.25}_{-0.18}$
γ_Σ	$-16.6^{+1.6}_{-1.8}$	$-15.8^{+1.4}_{-1.0}$
η_G	$0.263^{+0.038}_{-0.062}$	$0.194^{+0.012}_{-0.097}$
α_G	$6.15^{+0.58}_{-0.76}$	$9.9^{+1.0}_{-0.74}$
β_G	20 (fixed)	30 (fixed)
α_3	$-0.221^{+0.028}_{-0.027}$	$-0.217^{+0.027}_{-0.027}$
β_3	$2.43^{+0.11}_{-0.10}$	$2.40^{+0.11}_{-0.10}$
α_8	$0.36^{+0.19}_{-0.44}$	$0.43^{+0.11}_{-0.41}$
β_8	$3.37^{+0.63}_{-1.07}$	$3.51^{+0.42}_{-0.99}$
χ^2/ndf	233/219	234/219

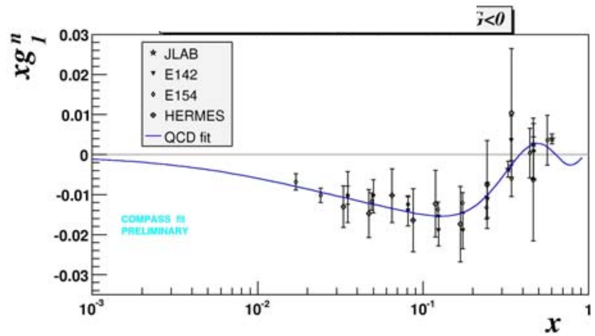
$\Delta G < 0$		
	Prog. Ref. [28]	Prog. Ref. [29]
η_Σ	0.321 ± 0.009	$0.329^{+0.009}_{-0.008}$
α_Σ	$1.39^{+0.15}_{-0.14}$	1.40 ± 0.12
β_Σ	$4.09^{+0.29}_{-0.27}$	$4.10^{+0.24}_{-0.23}$
γ_Σ	-	-
η_G	$-0.31^{+0.10}_{-0.14}$	$-0.181^{+0.042}_{-0.031}$
α_G	$0.39^{+0.64}_{-0.48}$	0.39 ± 0.17
β_G	$13.8^{+7.8}_{-5.3}$	$16.1^{+1.3}_{-4.0}$
α_3	-0.212 ± 0.027	$-0.208^{+0.027}_{-0.026}$
β_3	$2.44^{+0.11}_{-0.10}$	2.40 ± 0.10
α_8	0.42 ± 0.16	$0.347^{+0.071}_{-0.095}$
β_8	$3.53^{+0.56}_{-0.53}$	$3.31^{+0.30}_{-0.34}$
χ^2/ndf	247/219	248/219

FITTED xg_1 & WORLD DATA

The world data on $xg_1(x)$ at Q_0^2=3 GeV^2 are shown in this slide together with the QCD fit for $\Delta G < 0$ (blue lines).



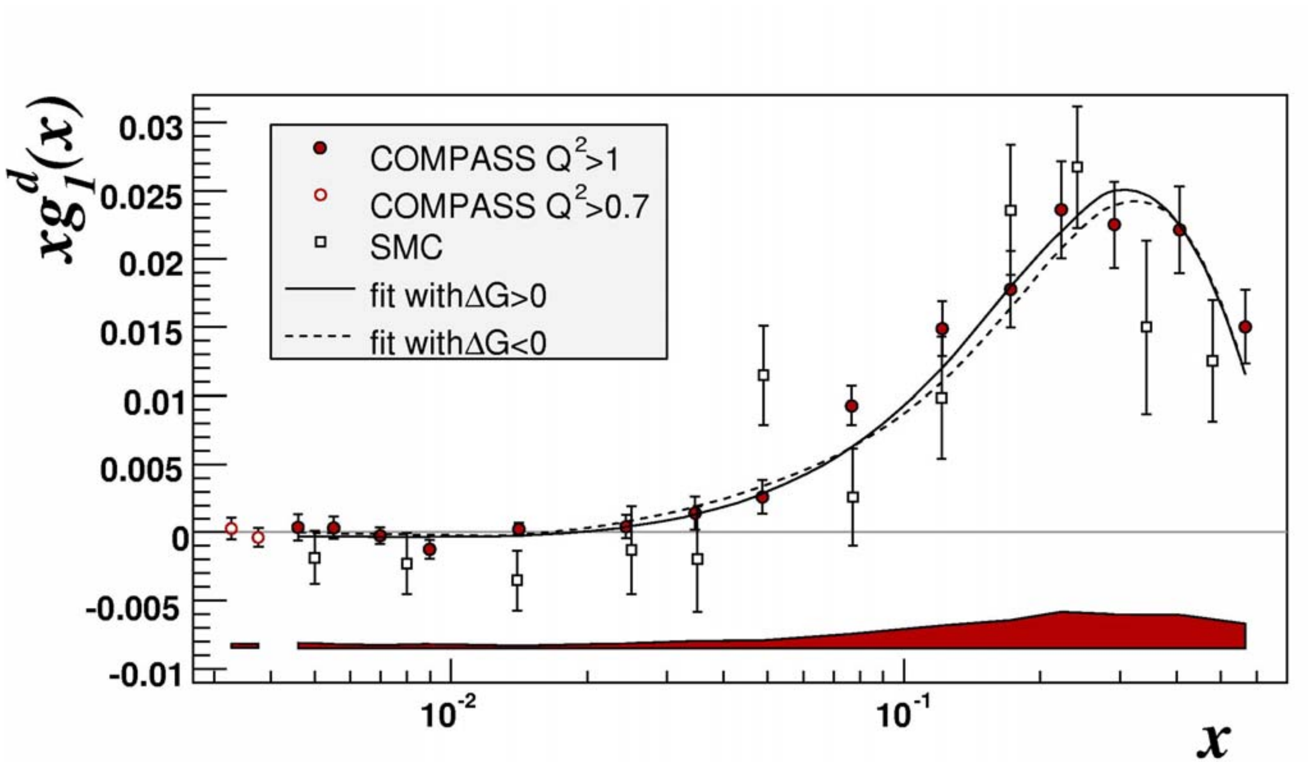




The fit reproduce trends of data rather well. But precisions of present measurements, especially for g_1^d and g_1^n, are still poor.

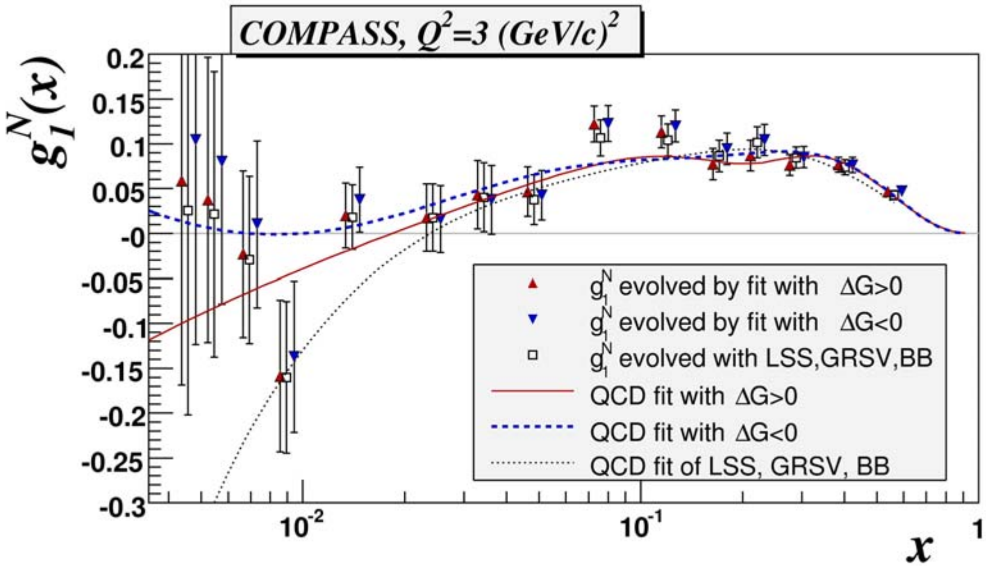
FITTED $xg_1^d(x)$ & NEW COMPASS DATA

Each of two solutions for PDF parameters is in agreement with new COMPASS data on g_1^d



FITTED AT $Q_0^2 = 3GeV^2$

The fitted g_1^N are compared with COMPASS data evolved to $Q_0^2 = 3GeV^2$ with ΔG > 0 and ΔG < 0, and with published PDF parametrizations*) obtained without new COMPASS measurements of g_1^d



-Even additional two points with $Q^2 > 0.7$ GeV^2 (due to large errors) do not help to choose between ΔG solutions,

-Previous parametrizations (averaged in above Fig.) do not reproduce the trend of COMPASS data at $x \to 0$,

-The fit with ΔG > 0 shows a dip at $x \approx 0.25$ related to the shape of ΔG(x)

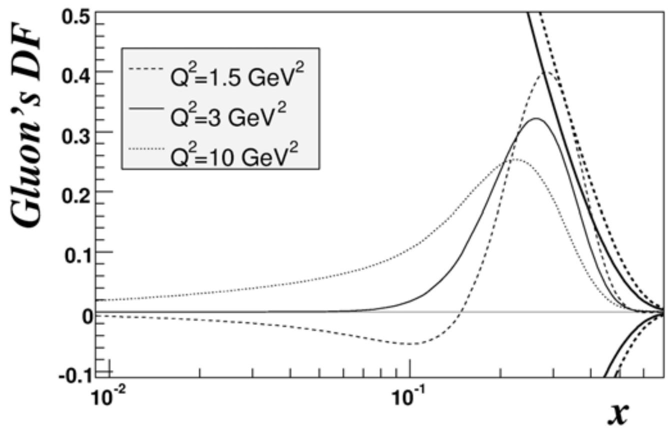
*) LSS = Leader, Sidorov, Stamenov, P.R. D73 (2006) 034023
GRSV = Glueck, Reya, Stratman, Vogelsang, P.R. D63 (2001) 094005
BB = Bluemlein, Boettcher, NP B636 (2002) 225

FITTED g_1^N AND SHAPE OF ΔG (x)

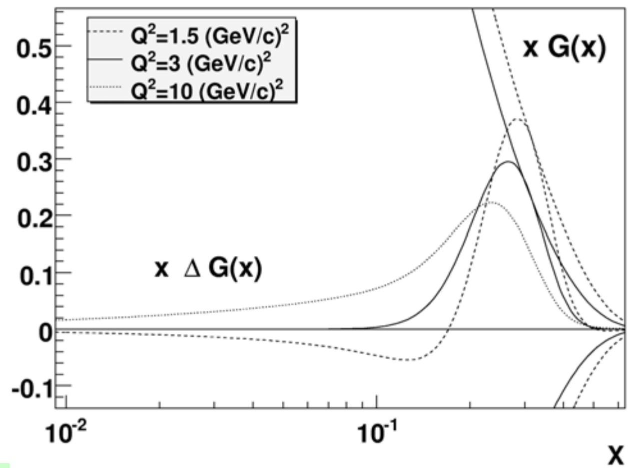
ΔG > 0

COMPASS data are compatible with positive ΔG (x). However in this case it must be close to zero at low *x*, to avoid pushing g_1^N down to negative values, and limited at higher *x* by positivity constraint $|\Delta G(x)| \leq G(x)$.

PROG 1



PROG 2



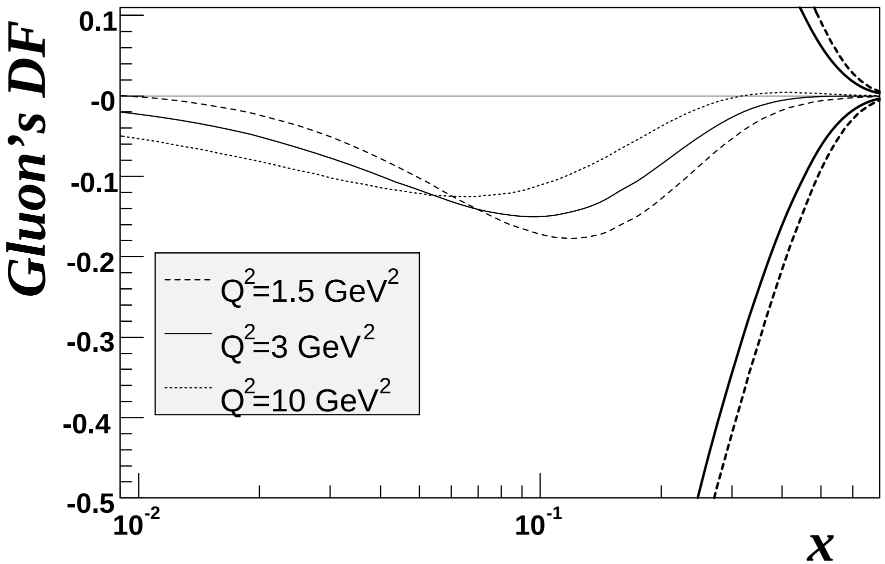
As a consequence, the whole ΔG(x) is squeezed in a narrow interval of *x* around the maximum at $x \sim \alpha_G/(\alpha_G+\beta_G) \approx 0.25$

FITTED g_1^N AND SHAPE OF ΔG (x), 2

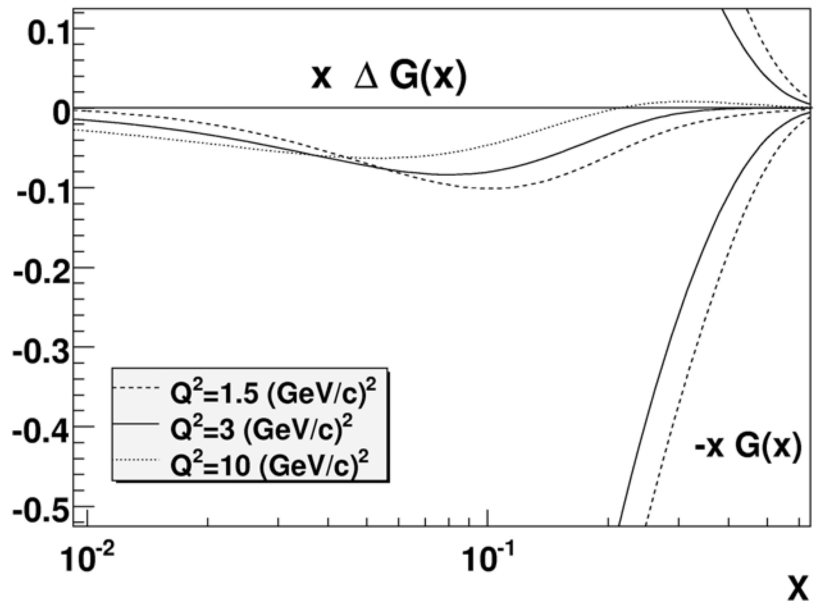
ΔG < 0

Fit with the negative ΔG (x) also reproduces well the COMPASS low x data.
But in this case the shape of ΔG (x) is rather smooth.

PROG 1



PROG 2



FIRST MOMENT OF ΔG(x)

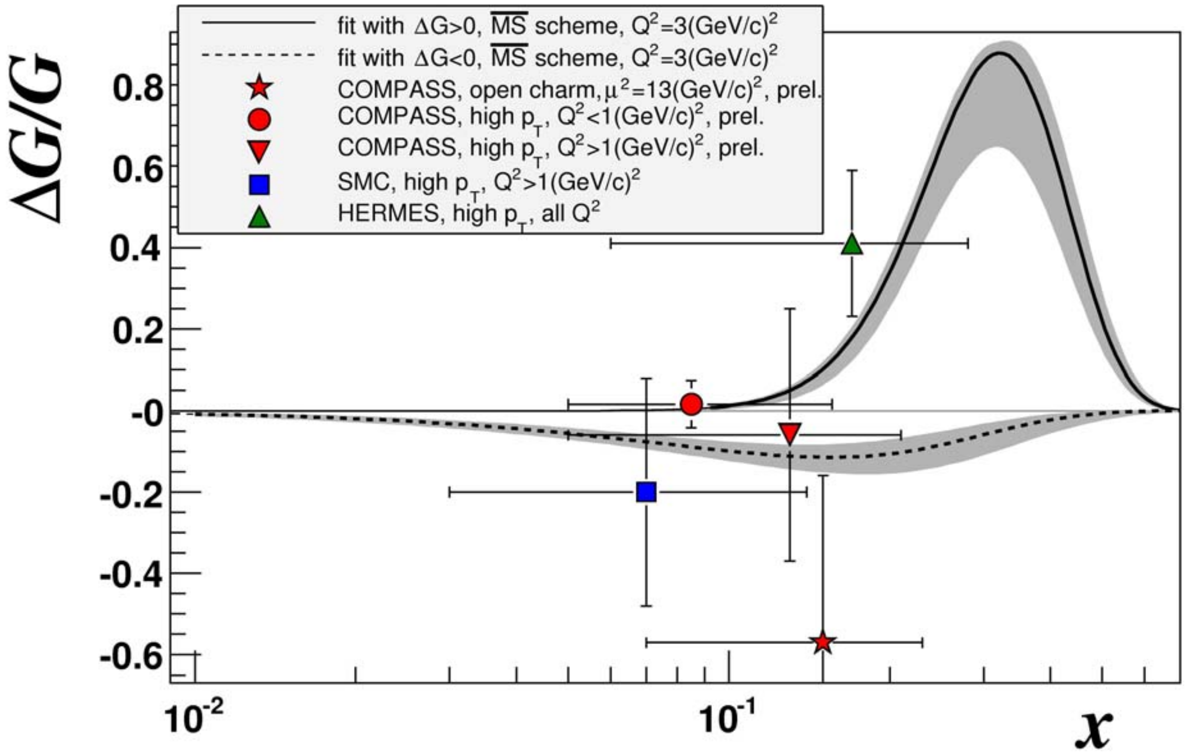
Although the gluon distributions strongly differ in two fits, their first moments are both small and about equal in absolute value (see Table 2):

$$\left|\eta_G\right| \approx 0.2 - 0.3$$

So, the gluon contribution to the SPIN of nucleons is rather small.

ΔG/G

The fitted $\Delta G^{(x)}/G^{(x)}$ are compared to direct measurement of ΔG/G



COMPASS high p_T, $Q^2 < 1$ GeV^2 point is in better agreement with

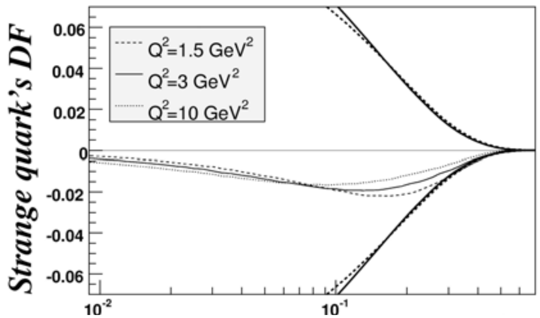
$\Delta G > 0$, although it is only 1.3σ away from $\Delta G < 0$.

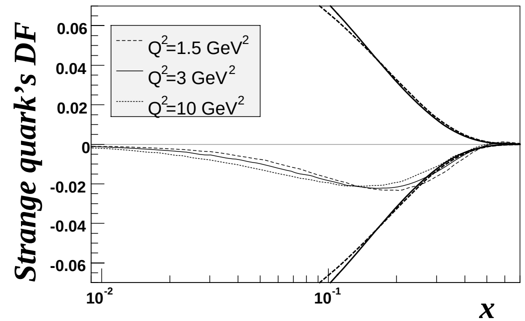
STRANGE QUARK DISTRIBUTIONS

The polarized strange quark distributions, obtained from $\Delta\Sigma(x)$ - $\Delta q_8(x)$ are almost identical for $\Delta G > 0$ and $\Delta G < 0$. They are negative and compatible with constraint $|\Delta S(x)| \leq s(x)$

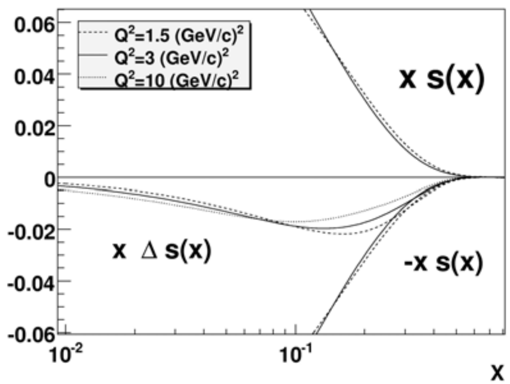
ΔG > 0 | **ΔG < 0**

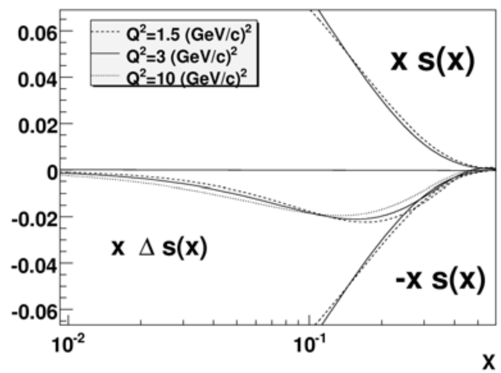
PROG 1





PROG 2





The strange quark polarization at $Q_0^2 = 3GeV^2$, found from fits, is

$$\left(\Delta s + \Delta \bar{s}\right)_{Q^2=3GeV^2} = -0.10 \pm 0.01(stat) \pm 0.01(evol.)$$

CONCLUSIONS

- New QCD NLO fits of the world g_1 data, including the latest COMPASS measurements of g_1^d, have been performed using two evolution formalisms.

- Fits have produced consistent results and yield two solutions for the PDF parameters with $\Delta G(x) > 0$ and $\Delta G(x) < 0$, which equally well describe the present g_1 data. The shapes of $\Delta G(x)$ are very different in two cases. Direct measurements of $\Delta G/G$, could help to choose between them.

- The first moments of the polarized gluon and strange quark distributions, found from fits at $Q_0^2 = 3GeV^2$, are equal to:

$$|\Delta G| \approx 0.2 - 0.3,$$

$$(\Delta s + \Delta \bar{s}) = -0.10 \pm 0.01(stat) + 0.01(evol)$$

OUTLOOK @ COMPASS

Further increase of statistics in 2006 and beyond

- Improvement in precision of direct $\Delta G/G$

  [$\sigma(\Delta G/G) \approx 0.045$ for high p_T, $Q^2 < 1$ GeV^2 pairs and

  $\approx$ 0.28 for open charm]

- Analysis of semi-inclusive hadron asymmetries in NLO approx

  (following suggestions in A.Sissakian, O.Shevchenko, O.Ivanov Phys.Rev. D73 (2006) 094026)