Hadron Tomography

Matthias Burkardt

`burkardt@nmsu.edu`

New Mexico State University

Las Cruces, NM, 88003, U.S.A.

Outline

- GPDs: probabilistic interpretation as Fourier transforms of impact parameter dependent PDFs
  - $H(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow q(x,\mathbf{b}_\perp)$
  - $\tilde{H}(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x,\mathbf{b}_\perp)$
  - $E(x,0,-\mathbf{\Delta}_\perp^2)$
    $\hookrightarrow \perp$ deformation of unpol. PDFs in $\perp$ pol. target
    - physics: orbital motion of the quarks

$\hookrightarrow$ intuitive explanation for SSAs
  - exclusive SSAs
  - Sivers effect

- $2\tilde{H}_T + E_T \longrightarrow \perp$ deformation of $\perp$ pol. PDFs in unpol. target
  - correlation between quark angular momentum and quark transversity

  $\hookrightarrow$ Boer-Mulders function $h_1^\perp(x,\mathbf{k}_\perp)$

- Summary

Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$\begin{aligned} \int dx H_q(x,\xi,t) &= F_1^q(t) & \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t) \\ \int dx E_q(x,\xi,t) &= F_2^q(t) & \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t), \end{aligned}$$

  - x_i and x_f are the momentum fractions of the quark before and after the momentum transfer
  - $2\xi = x_f - x_i$
- GPDs can be probed in deeply virtual Compton scattering (DVCS)

Generalized Parton Distributions (GPDs)

- formal definition (unpol. quarks):

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p)$$

$$+E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

- in the limit of vanishing t and ξ, the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

$$H_q(x,0,0) = q(x) \qquad \tilde{H}_q(x,0,0) = \Delta q(x).$$

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,\xi,t)$	?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,0,t)$	$q(x,\mathbf{b}_\perp)$

$q(x,\mathbf{b}_\perp)$ = impact parameter dependent PDF

Impact parameter dependent PDFs

- define $\perp$ localized state [D.Soper,PRD**15**, 1141 (1977)]

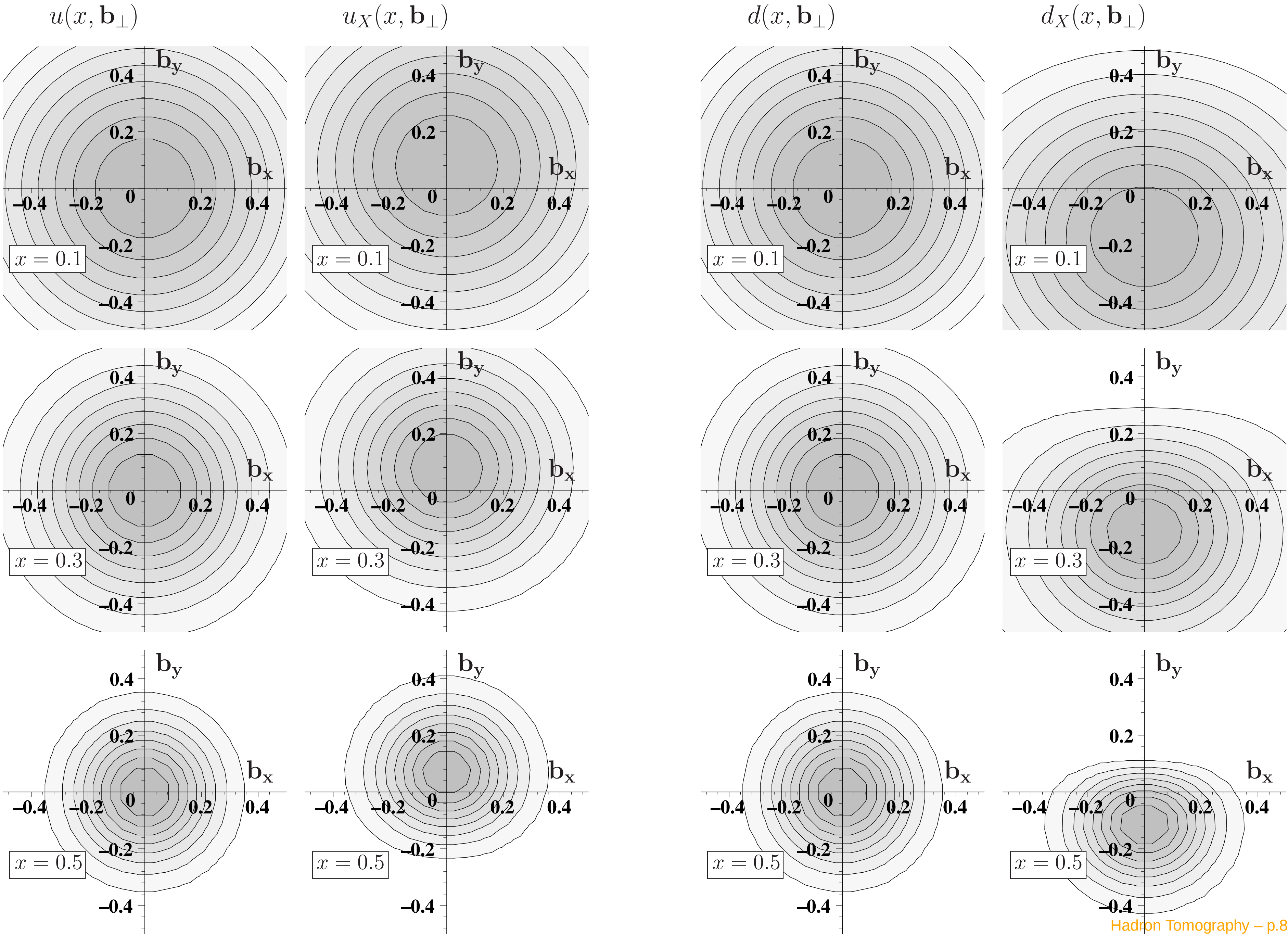
$$\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\lambda\right\rangle \equiv \mathcal{N}\int d^2\mathbf{p}_\perp \left|p^+,\mathbf{p}_\perp,\lambda\right\rangle$$

Note: $\perp$ boosts in IMF form Galilean subgroup ⇒ this state has
$\mathbf{R}_\perp \equiv \frac{1}{P^+}\int dx^- d^2\mathbf{x}_\perp\, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
(cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x,\mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi}\left\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right|\bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp)\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$$\hookrightarrow \boxed{\begin{aligned} q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} H(x,0,-\mathbf{\Delta}_\perp^2), \\ \Delta q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} \tilde{H}(x,0,-\mathbf{\Delta}_\perp^2), \end{aligned}}$$


$u(x,\mathbf{b}_\perp)$
$u_X(x,\mathbf{b}_\perp)$
$d(x,\mathbf{b}_\perp)$
$d_X(x,\mathbf{b}_\perp)$
$\mathbf{b_y}$
$\mathbf{b_x}$
−0.4
−0.2
0
0.2
0.4
$x = 0.1$
$x = 0.3$
$x = 0.5$

Transversely Deformed Distributions and $E(x,0,-\mathbf{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\uparrow\rangle \;=\; H(x,0,-\mathbf{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\downarrow\rangle \;=\; -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

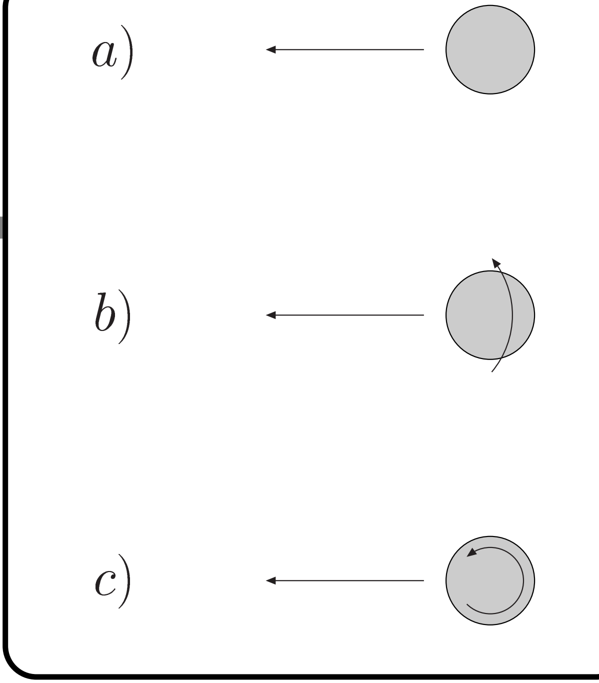
- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+,\mathbf{R}_\perp = \mathbf{0}_\perp,\uparrow\rangle + |p^+,\mathbf{R}_\perp = \mathbf{0}_\perp,\downarrow\rangle$.

↪ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 !
  [X.Ji, PRL **91**, 062001 (2003)]

physical origin for $\perp$ distortion



- Consider nucleon moving in $\hat{z}$-direction.
- quarks orbiting around the axis of motion (long. pol. nucleon), the orbital motion does not affect the longitudinal momentum distribution.
- quarks orbiting around $\perp$ direction ($\perp$ pol.nucleon) orbital motion adds/subtracts to long. momentum for $y > 0$ and $y < 0$ respectively
- PDFs rapidly fall with x

↪ boost/de-boost on $\pm\hat{y}$ side results in enhancement/suppression of $q(x, \mathbf{b}_\perp)$.

- details described by $E(x, 0, -\boldsymbol{\Delta}_\perp^2)$.

Transversely Deformed Distributions and $E(x,0,-\boldsymbol{\Delta}_\perp^2)$

- $q(x,\mathbf{b}_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons !
- mean $\perp$ deformation of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x,\mathbf{b}_\perp) b_y = \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q^p}{2M}$$

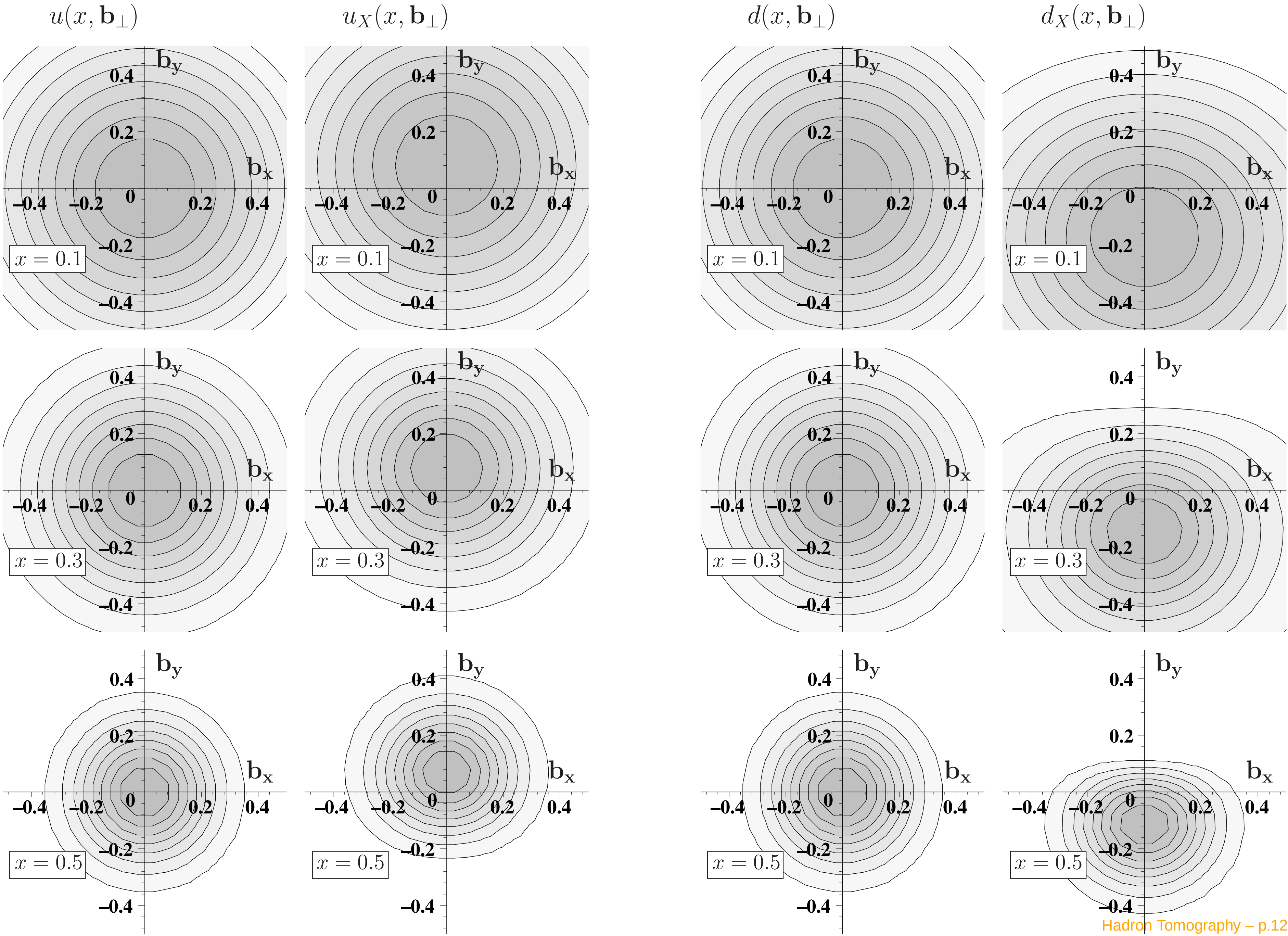
with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2fm)$

- simple model: for simplicity, make ansatz where $E_q \propto H_q$

$$\begin{aligned} E_u(x,0,-\boldsymbol{\Delta}_\perp^2) &= \frac{\kappa_u^p}{2} H_u(x,0,-\boldsymbol{\Delta}_\perp^2) \\ E_d(x,0,-\boldsymbol{\Delta}_\perp^2) &= \kappa_d^p H_d(x,0,-\boldsymbol{\Delta}_\perp^2) \end{aligned}$$

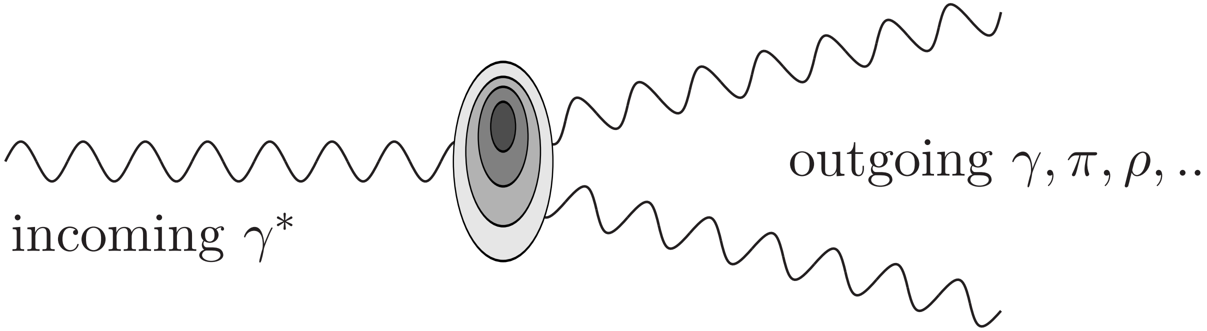
with $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.$

- Model too simple but illustrates that anticipated deformation is very significant since κ_u and κ_d known to be large!


$u(x,\mathbf{b}_\perp)$
$u_X(x,\mathbf{b}_\perp)$
$d(x,\mathbf{b}_\perp)$
$d_X(x,\mathbf{b}_\perp)$
$x = 0.1$
$x = 0.3$
$x = 0.5$
$\mathrm{b_x}$
$\mathrm{b_y}$
−0.4
−0.2
0
0.2
0.4

Exclusive SSAs

(A.Belitsky & D.Müller; see also S.J.Brodsky & A.Mukherjee)



- For simplicity, only $\perp$ momentum transfer

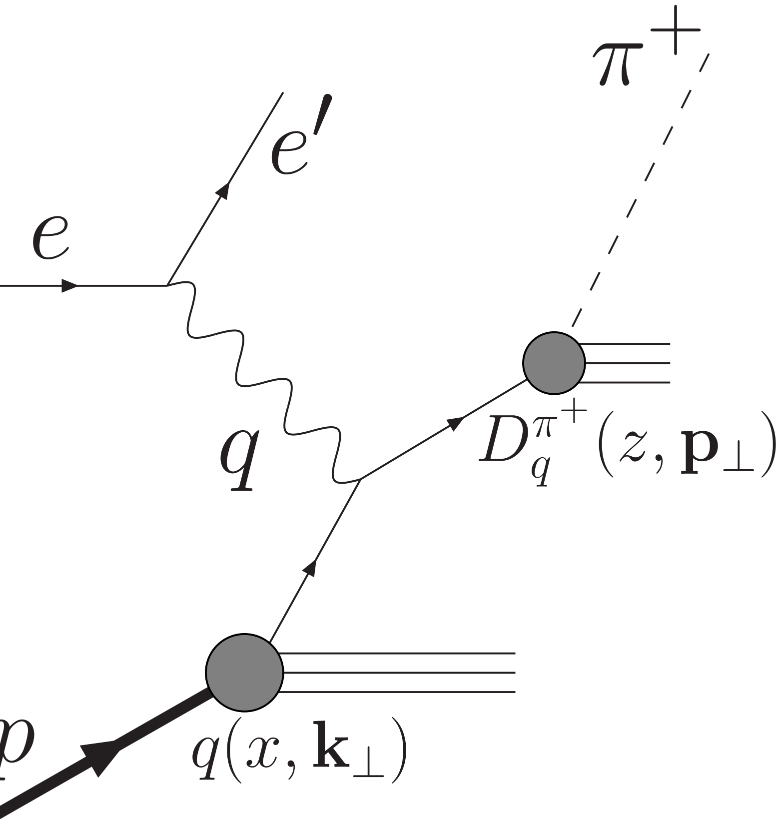
$$\mathcal{A} \propto \int d^2\mathbf{b}_\perp \, q(\mathbf{b}_\perp, x) T_q e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp}$$

where T_q is the parton-level “Compton”-amplitude

$\hookrightarrow$ left-right asymmetry of quark distribution translates directly into left-right asymmetry of the scattering amplitude

$\hookrightarrow$ exclusive SSA probe GPDs which describe this deformation (“Huygen’s principle!”)

SSAs in SIDIS ($\gamma + p\uparrow \longrightarrow \pi^+ + X$)



- use factorization (high energies) to express momentum distribution of outgoing π^+ as convolution of
  - momentum distribution of quarks in nucleon

  $\hookrightarrow$ unintegrated parton density $f_{q/p}(x, \mathbf{k}_\perp)$
  - momentum distribution of π^+ in jet created by leading quark q

  $\hookrightarrow$ fragmentation function $D_q^{\pi^+}(z, \mathbf{p}_\perp)$

- average $\perp$ momentum of pions obtained as sum of
  - average $\mathbf{k}_\perp$ of quarks in nucleon (Sivers effect)
  - average $\mathbf{p}_\perp$ of pions in quark-jet (Collins effect)

GPD $\longleftrightarrow$ SSA (Sivers)

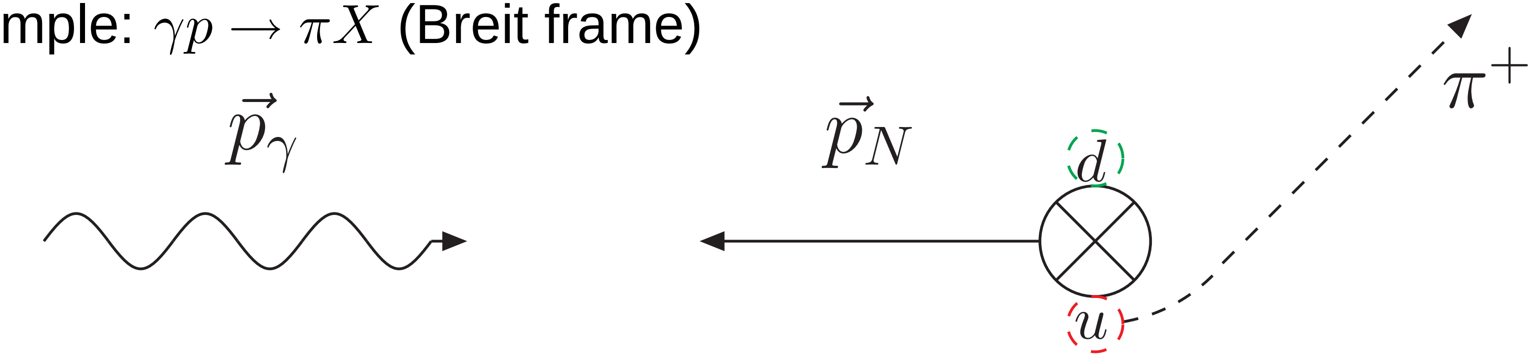
- Sivers: distribution of **unpol.** quarks in $\perp$ pol. proton

$$f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot S}{M}$$

- without FSI, $\langle\mathbf{k}_\perp\rangle = 0$, i.e. $f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2) = 0$
- with FSI, $\langle\mathbf{k}_\perp\rangle \neq 0$ (Brodsky, Hwang, Schmidt)
- FSI formally included by appropriate choice of Wilson line gauge links in gauge invariant def. of $f_{q/p}(x,\mathbf{k}_\perp)$
- What should we expect for Sivers effect in QCD ?

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \to \pi X$ (Breit frame)



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign “determined” by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

↪ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

↪ correlation between sign of κ_q^p and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q^p$

- $f_{1T}^{\perp q} \sim -\kappa_q^p$ confirmed by HERMES results (also consistent with COMPASS $f_{1T}^{\perp u} + f_{1T}^{\perp q} \approx 0$)

Chirally Odd GPDs

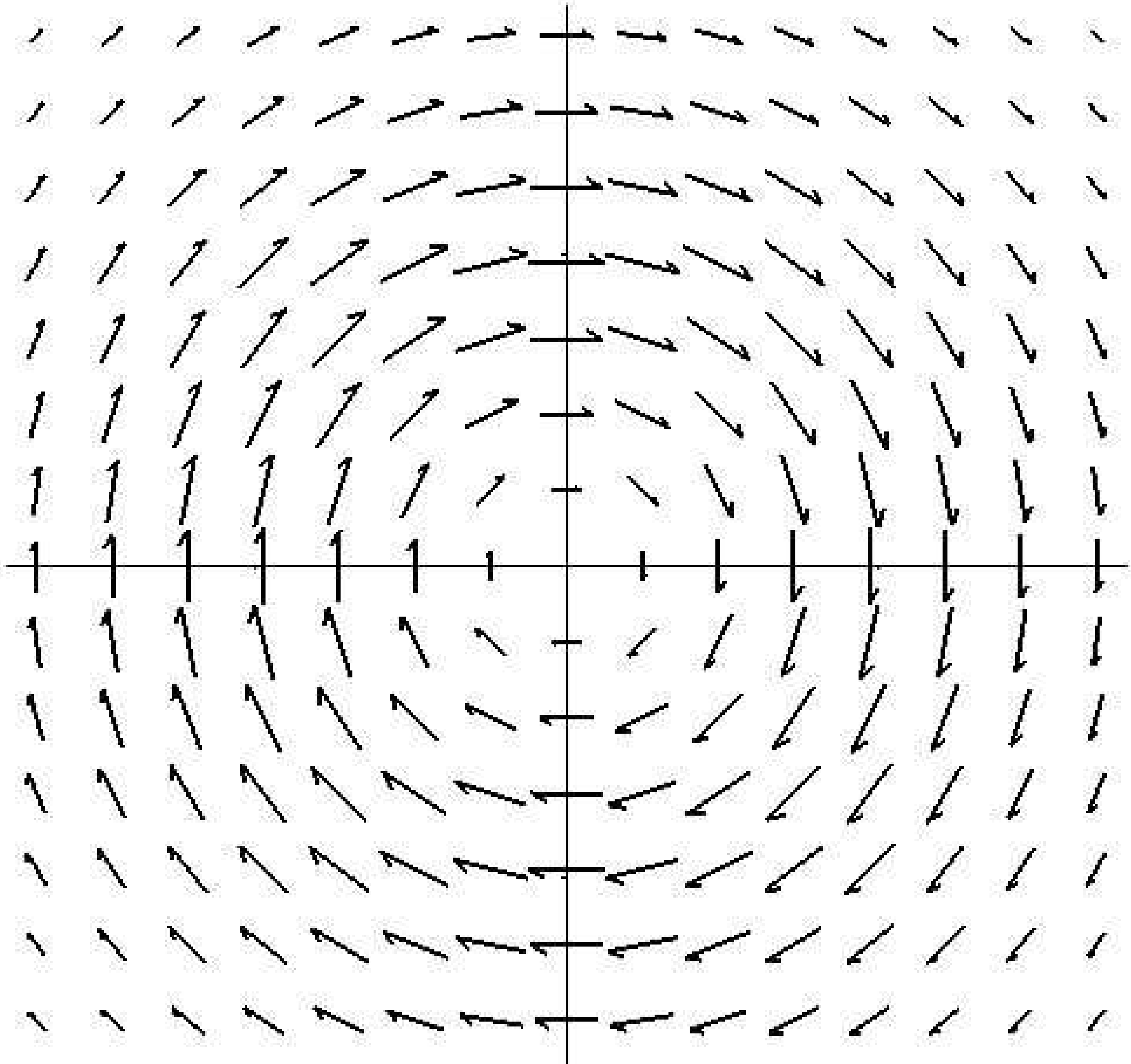
$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \sigma^{+j}\gamma_5 q\left(\frac{x^-}{2}\right) \right| p \right\rangle = \quad H_T \bar{u}\sigma^{+j}\gamma_5 u + \tilde{H}_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha P_\beta}{M^2} u$$
$$+ E_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha \gamma_\beta}{2M} u + \tilde{E}_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}P_\alpha \gamma_\beta}{M} u$$

- See also M.Diehl+P.Hägler, hep-ph/0504175.
- Fourier trafo of $\bar{E}_T^q \equiv 2\tilde{H}_T^q + E_T^q$ for $\xi = 0$ describes distribution of transversity for unpolarized target in $\perp$ plane

$$q^i(x, \mathbf{b}_\perp) = \frac{\varepsilon^{ij}}{2M}\frac{\partial}{\partial b_j}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} \bar{E}_T^q(x, 0, -\mathbf{\Delta}_\perp^2)$$

- origin: correlation between quark spin (i.e. transversity) and angular momentum

Transversity Distribution in Unpolarized Target



Chirally Odd GPDs

- $J^i = \frac{1}{2}\varepsilon^{ijk}\int d^3x \left[T^{0j}x^k - T^{0k}x^j\right]$
- J_q^x diagonal in transversity, projected with $\frac{1}{2}(1 \pm \gamma^x\gamma_5)$, i.e. one can decompose

$$J_q^x = J_{q,+\hat{x}}^x + J_{q,-\hat{x}}^x$$

where $J_{q,\pm\hat{x}}^x$ is the contribution (to J_q^x) from quarks with positive (negative) transversity

↪ derive relation quantifying the correlation between ⊥ quark spin and angular momentum [M.B., PRD72, 094020 (2006); PLB639, 462 (2006)]

$$\left\langle J_{q,+\hat{y}}^y \right\rangle = \frac{1}{4}\int dx \left[H_T^q(x,0,0) + \bar{E}_T^q(x,0,0)\right] x$$

(note: this relation is not a decomposition of J_q into transversity and orbital)

Boer-Mulders Function

- SIDIS: attractive FSI expected to convert position space asymmetry into momentum space asymmetry

↪ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction

↪ (qualitative) connection between Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$ and the chirally odd GPD $\bar{E}_T$ that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x, \mathbf{k}_\perp)$ and the GPD E.

- Boer-Mulders: distribution of $\perp$ **pol.** quarks in **unpol.** proton

$$f_{q^\uparrow/p}(x, \mathbf{k}_\perp) = \frac{1}{2}\left[f_1^q(x, \mathbf{k}_\perp^2) - h_1^{\perp q}(x, \mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp) \cdot S_q}{M}\right]$$

- $h_1^{\perp q}(x, \mathbf{k}_\perp^2)$ can be probed in DY (RHIC, J-PARC, GSI) and tagged SIDIS (JLab, eRHIC), using Collins-fragmentation

Boer-Mulders Function

- Model calculations (bag model, const. quark model, NJL-model) indicate:
  - $\bar{E}_T > 0$ for u and d quarks in nucleon and pion, indicating a "universal" spin-orbit correlation for valence quarks
  - $\bar{E}_T > E^u$, i.e. stronger correlation between L_q and quark spin than between L_q and the nucleon spin
- confirmed by lattice calculations (P.Hägler et al.)

↪ several interesting predictions:

  - $h_1^\perp(x, \mathbf{k}_\perp)$ same sign (negative) as $f_{1T}^\perp(x, \mathbf{k}_\perp)$
  - "universal sign" for valence $h_1^\perp$
  - $\left|h_1^\perp\right| > \left|f_{1T}\right|$

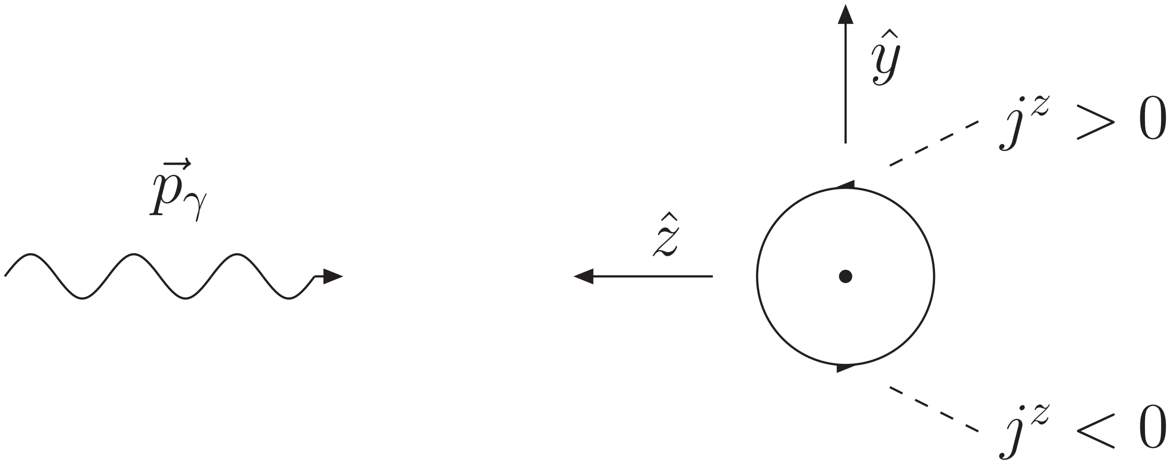
↪ let's measure $h_1^\perp$ to learn more about spin-orbit correlations for quarks!

Summary

- GPDs $\xrightarrow{FT}$ PDFs in impact parameter space
- $E(x,0,-\boldsymbol{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target
  - $\hookrightarrow$ origin for deformation: orbital motion of the quarks
  - $\hookrightarrow$ simple mechanism (attractive FSI) to predict sign of f_{1T}^q
- distribution of $\perp$ polarized quarks in unpol. target described by chirally odd GPD $\bar{E}_T^q = 2\bar{H}_T^q + \tilde{E}_T^q$
  - $\hookrightarrow$ origin: correlation between orbital motion and spin of the quarks
  - $\hookrightarrow$ attractive FSI $\Rightarrow$ measurement of $h_1^\perp$ (DY,SIDIS) provides information on $\bar{E}_T^q$ and hence on spin-orbit correlations

Intuitive connection with $\vec{L}_q$

- Electromagnetic interaction couples to vector current. Due to kinematics of the DIS-reaction (and the choice of coordinates — $\hat{z}$-axis in direction of the momentum transfer) the virtual photons “see” (in the Bj-limit) only the $j^+ = j^0 + j^z$ component of the quark current
- If up-quarks have positive orbital angular momentum in the $\hat{x}$-direction, then j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



Intuitive connection with $\vec{L}_q$

- Electromagnetic interaction couples to vector current. Due to kinematics of the DIS-reaction (and the choice of coordinates — $\hat{z}$-axis in direction of the momentum transfer) the virtual photons “see” (in the Bj-limit) only the $j^+ = j^0 + j^z$ component of the quark current
- If up-quarks have positive orbital angular momentum in the $\hat{x}$-direction, then j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side

$\hookrightarrow$ j^+ is deformed not because there are more quarks on one side than on the other but because the DIS-photons (coupling only to j^+) “see” the quarks on the $+\hat{y}$ side better than on the $-\hat{y}$ side.

- $\perp$ deformation described by $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$

$\hookrightarrow$ not surprising to find that $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$ enters the Ji relation

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x, 0, 0) + E_q(x, 0, 0) \right] x.$$

back

$\perp$ Single Spin Asymmetry (Sivers)

- Naive definition of unintegrated parton density

$$f(x,\mathbf{k}_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip\cdot\xi} \left\langle P,S\left|\bar{q}(0)\gamma^+ q(\xi)\right|P,S\right\rangle\Big|_{\xi^+=0}.$$

- Time-reversal invariance $\Rightarrow f(x,\mathbf{k}_\perp) = f(x,-\mathbf{k}_\perp)$

$\hookrightarrow$ Asymmetry $\int d^2\mathbf{k}_\perp f(x,\mathbf{k}_\perp)\,\mathbf{k}_\perp = 0$

- Same conclusion for gauge invariant definition with straight Wilson line $U_{[0,\xi]} = P\exp\left(ig\int_0^1 ds\xi_\mu A^\mu(s\xi)\right)$

$\perp$ Single Spin Asymmetry (Sivers)

- Naively (time-reversal invariance) $f(x, \mathbf{k}_\perp) = f(x, -\mathbf{k}_\perp)$
- However, including the final state interaction (FSI) results in nonzero asymmetry of the ejected quark! (Brodsky, Hwang, Schmidt)
- Gauge invariant definition requires quark to be connected by gauge link. Choice of path not arbitrary but must be chosen along path of outgoing quark to incorporate FSI

$$f(x, \mathbf{k}_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip\cdot\xi} \left\langle P, S \left| \bar{q}(0) U_{[0,\infty]} \gamma^+ U_{[\infty,\xi]} q(\xi) \right| P, S \right\rangle \Big|_{\xi^+=0}$$

with $U_{[0,\infty]} = P \exp\left(ig \int_0^\infty d\eta^- A^+(\eta)\right)$

Sivers Mechanism in $A^+ = 0$ gauge

- Gauge link along light-cone trivial in light-cone gauge

$$U_{[0,\infty]} = P \exp\left(ig \int_0^\infty d\eta^- A^+(\eta) \right) = 1$$

↪ Puzzle: Sivers asymmetry seems to vanish in LC gauge (time-reversal invariance)!

- X.Ji: fully gauge invariant definition for $P(x, \mathbf{k}_\perp)$ requires additional gauge link at $x^- = \infty$

$$\begin{aligned} f(x, \mathbf{k}_\perp) &= \int \frac{dy^- d^2\mathbf{y}_\perp}{16\pi^3} e^{-ixp^+y^- + i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \\ &\times \left\langle p, s \left| \bar{q}(y) \gamma^+ U_{[y^-, \mathbf{y}_\perp; \infty^-, \mathbf{y}_\perp]} U_{[\infty^-, \mathbf{y}_\perp, \infty^-, \mathbf{0}_\perp]} U_{[\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]} q(0) \right| p, s \right\rangle \end{aligned}$$

back