The constraint on the spin dependent structure function g_1 at low Q^2 through the sum rule corresponding to the moment at $n = 0$

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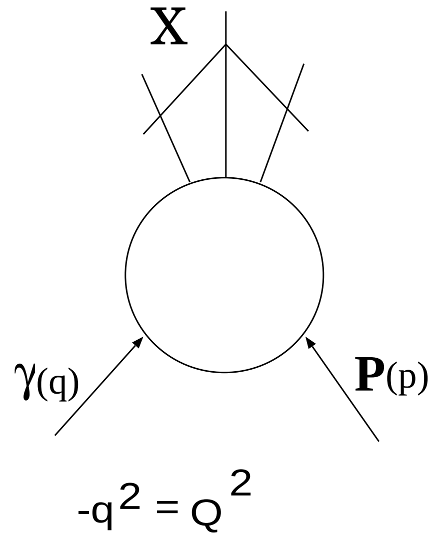
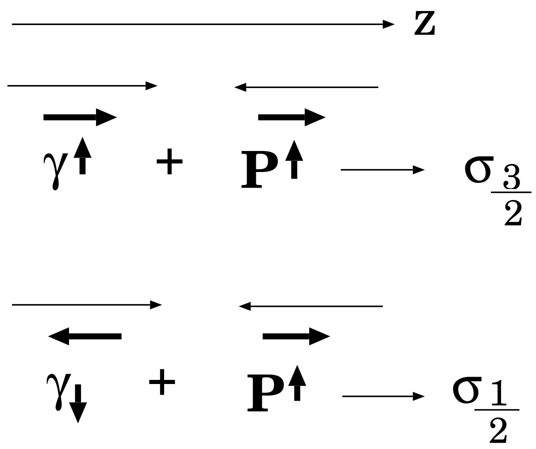
(1)Kinematics





$\gamma\uparrow + q\downarrow \longrightarrow q\uparrow$

$\gamma\uparrow + q\uparrow \longrightarrow \times$

$\sigma_{\frac{3}{2}} \sim \gamma\uparrow \mathbf{P}\uparrow \sim \Sigma e_i^2 \mathbf{q}\downarrow$

$\gamma\downarrow + q\uparrow \longrightarrow q\downarrow$

$\gamma\downarrow + q\downarrow \longrightarrow \times$

$\sigma_{\frac{1}{2}} \sim \gamma\downarrow \mathbf{P}\uparrow \sim \Sigma e_i^2 \mathbf{q}\uparrow$

$\sigma_{\frac{3}{2}} - \sigma_{\frac{1}{2}} \sim \Sigma(e_i^2 \mathbf{q}\downarrow - e_i^2 \mathbf{q}\uparrow) \sim - g_1$

$$W^{ab}_{\mu\nu}|_{spin} = \frac{1}{4\pi}\int d^4x \exp(iqx) < p, s|J^a_\mu(x)\cdot J^b_\nu(0)|p, s >_c |_{spin}$$

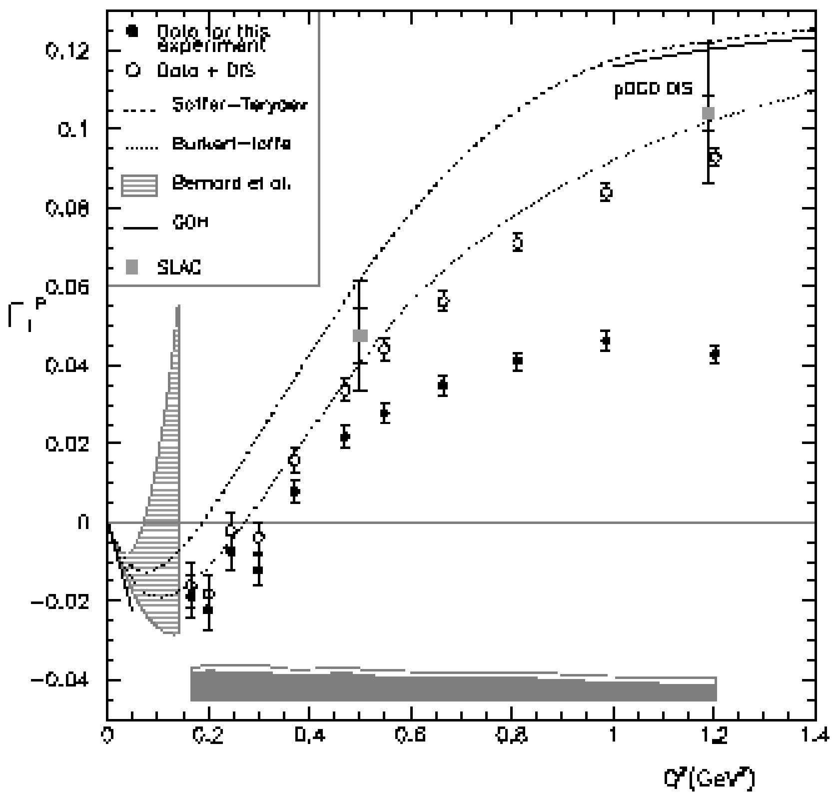
$$= i\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma G^{ab}_1 + i\epsilon_{\mu\nu\lambda\sigma}q^\lambda(\nu s^\sigma - q\cdot sp^\sigma)G^{ab}_2$$

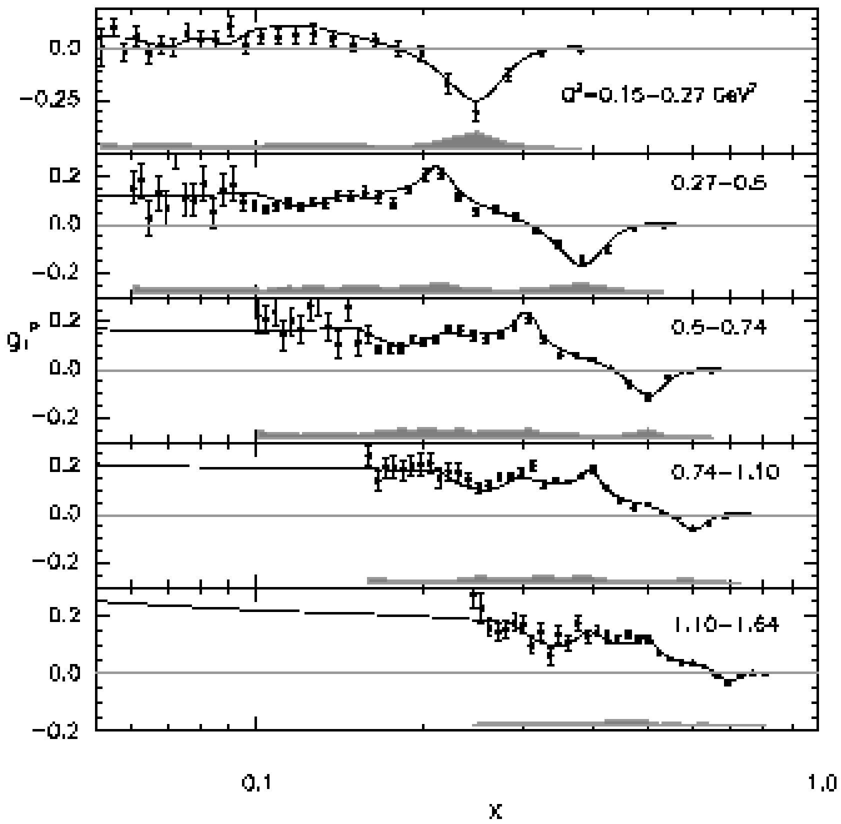
$$I^p(Q^2) = \frac{2m_p^2}{Q^2}\Gamma^P(Q^2)$$
$$\Gamma^P(Q^2) = \int_0^1 dx g_1^p(x, Q^2)$$

At $Q^2 = 0$, we have

$$\begin{aligned} I^p(0) &= -\frac{m_p^2}{8\pi^2\alpha}\int \frac{d\nu}{\nu}\{\sigma^p_{3/2}(\nu) - \sigma^p_{1/2}(\nu)\} \\ &= -\frac{\kappa_p^2}{4} < 0 \end{aligned}$$

On the other hand, at large Q^2, we have $\Gamma^P(Q^2) > 0$. This is because the $\Delta(1232)$ gives large negative contribution in the small Q^2 region. Experimentally, at CLAS(R.Fatemi et al .PRL91.(2003)222002), $\Gamma^P(Q^2)$ is studied and shown to change sign near $Q^2 = 0.3(GeV/c)^2$.





R.Fatemi et al .PRL91.(2003)222002

(2)Sum rules based on the canonical quantization on the null-plane

$$q^{(\pm)}(x) = \Lambda^{\pm} q(x) \quad \Lambda^{\pm} = \frac{1}{2}(1 \pm \gamma^0 \gamma^3) \quad x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$$

$$\{q^{(+)\dagger}(x), q^{(+)}(0)\}|_{x^+=0} = \sqrt{2}\Lambda^+ \delta^2(\vec{x}^{\perp})\delta(x^-)$$

$$J_a^+(x) = \bar{q}(x)\gamma^+ \frac{\lambda_a}{2} q(x) = \sqrt{2} q^{(+)\dagger}(x) \frac{\lambda_a}{2} q^{(+)}(x)$$

$$Q_a = \int d^2x^{\perp} dx^- J_a^+(x)|_{x^+=0}$$

$$J_a^i(x) = q^{(+)\dagger}(x)\gamma^0\gamma^i\frac{\lambda_a}{2}q^{(-)}(x) + q^{(-)\dagger}(x)\gamma^0\gamma^i\frac{\lambda_a}{2}q^{(+)}(x)$$

This current depends on $q^{(-)}(x)$ which can be expressed by the $q^{(+)}(x)$ through the equation of motion. In this sense, the model dependence enters, however, the following commutation relation holds in QCD.

$$\begin{aligned}
[J_a^+(x), J_b^i(0)]|_{x^+=0} &= i[s^{+\beta i\alpha}\partial_\alpha[\Delta(x)G_{c\beta}(x|0)] \\
&- 2g^{+\alpha}g^{i\beta}\partial_\alpha[\Delta(x)]G_{c\beta}(x|0) \\
&- \epsilon^{+i\alpha\beta}\partial_\alpha[\Delta(x)G^5_{c\beta}(x|0)]]
\end{aligned}$$

$$\Delta(x)|_{x^+=0} = -\frac{1}{4}\epsilon(x^-)\delta^2(\vec{x}^\perp)$$
$$G_c^\beta(x|0) = d_{abc}A_c^\beta(x|0) + f_{abc}S_c^\beta(x|0)$$
$$G_c^{5\beta}(x|0) = d_{abc}S_c^{5\beta}(x|0) - f_{abc}A_c^{5\beta}(x|0)$$

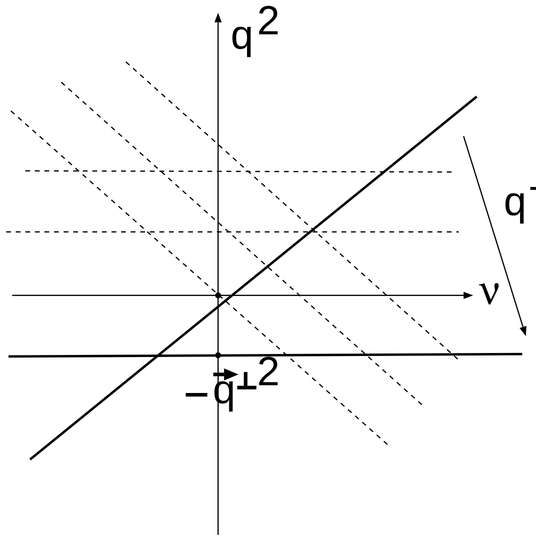
The sum rule for the spin dependent function g_1 can be derived from this relation.

(3)The basic assumption to derive the sum rule

$$C_{ab}(p\cdot q, q^2) = \int d^4x \exp(iqx) < p|[J_a(x), J_b(0)]|p >_c .$$

$$\int_{-\infty}^{\infty} dq^- C_{ab}(p\cdot q, q^2)$$

$$= \quad 2\pi \int d^4x\delta(x^+) \exp(iq^+x^- - i\vec{q}^{\perp}\vec{x}^{\perp}) < p|[J_a(x), J_b(0)]|p >_c$$



Change variable from q^- *to* $\nu = p^+q^- + p^-q^+ - \vec{p}^{\perp}\vec{q}^{\perp}$.
Then we obtain $q^2 = 2q^+\frac{\nu - p^-q^+ + \vec{p}^{\perp}\vec{q}^{\perp}}{p^+} - \vec{q}^{\perp 2}$.
To set $q^+ = 0$ before ν integration, we need the superconvergence relation to neglect the contributions from the time-like q^2 region.

(4)The sum rule for the g_1 in the isovector reaction

$$\int_0^1 \frac{dx}{x} g_1^{[ab]}(x, Q^2) = -\frac{1}{16} f_{abc} \int_{-\infty}^{\infty} d\alpha [A_c^5(\alpha, 0) + \alpha \bar{A}_c^5(\alpha, 0)]$$

$$\begin{aligned} < p, s | A_c^{5\beta}(x|0) | p, s >_c &= s^\mu A_c^5(p \cdot x, x^2) + p^\mu (x \cdot s) \bar{A}_c^5(p \cdot x, x^2) \\ &+ x^\mu (x \cdot s) \tilde{A}_c^5(p \cdot x, x^2) \end{aligned}$$

Since the right-hand side is Q^2 independent, we obtain for the anti-symmetric combination with respect to a, b

$$\int_0^1 \frac{dx}{x} g_1^{[ab]}(x, Q^2) = \int_0^1 \frac{dx}{x} g_1^{[ab]}(x, Q_0^2)$$

Now, we take $Q_0^2 = 0$ and use the relation

$$G_1^{ab}(\nu, 0) = -\frac{1}{8\pi^2 \alpha_{em}} \{\sigma_{3/2}^{ab}(\nu) - \sigma_{1/2}^{ab}(\nu)\} = -\frac{1}{8\pi^2 \alpha_{em}} \Delta\sigma^{ab}(\nu)$$

By setting $a = (1+i2)/\sqrt{2}, b = a^\dagger$, and separating out the elastic contribution, we obtain the sum rule which relates the g_1 and the cross section of the photo-production in the isovector reactions.
When the integral diverges we rewrite the sum rule as follows.

$$\int_0^1 \frac{dx}{x}\{g_1(x,Q^2) - f(x,Q^2)\} + \int_0^1 \frac{dx}{x} f(x,Q^2)$$
$$= \int_0^1 \frac{dx}{x}\{g_1(x,Q_0^2) - f(x,Q_0^2)\} + \int_0^1 \frac{dx}{x} f(x,Q_0^2)$$

where we set

$$f(x,Q^2) = \beta(Q^2)x^{-\alpha(0,\epsilon)} + f_1(x,Q^2) \qquad \alpha(0,\epsilon) = a - \epsilon$$

We take the limit $\epsilon \to a$ from the region above a.

$$\int_0^1 \frac{dx}{x} f(x,Q^2) = \frac{\beta(Q^2)}{\epsilon - a} + \int_0^1 \frac{dx}{x} f_1(x,Q^2)$$

After taking out the pole term from both-hand side of the sum rule, we take $\epsilon \rightarrow 0$.

$$\int_0^1 \frac{dx}{x}\{g_1(x,Q^2) - f(x,Q^2)\}$$
$$= \int_0^1 \frac{dx}{x}\{g_1(x,Q_0^2) - f(x,Q_0^2)\} + \int_0^1 \frac{dx}{x}\{f(x,Q_0^2) - f(x,Q^2)\}$$

Now we take $f(x,Q^2) = g_1(x,Q^2)$ above $\nu_c^Q = m_p E_Q$ where $E_Q = E_c + Q^2/2m_p$ and $f(x,Q^2) = 0$ below it, where E is a energy in the laboratory frame. Then the sum rule can be transformed as

$$B(Q^2) + K(E_c, Q^2) = \int_{E_0}^{E_Q} \frac{dE}{E}[2g_1^{1/2}(x,Q^2) - g_1^{3/2}(x,Q^2)] + \frac{m_p}{8\pi^2\alpha_{em}} \int_{E_0}^{E_c} dE[2\Delta\sigma^{1/2} - \Delta\sigma^{3/2}]$$

$$B(Q^2) = \frac{1}{4}\{(\mu_p - \mu_n) - \frac{1}{1+Q^2/4m_p^2} G_M^+(Q^2)[G_E^+(Q^2) + \frac{Q^2}{4m_p^2} G_M^+(Q^2)]\}$$

$$G_E^+(Q^2) = G_E^p(Q^2) - G_E^n(Q^2) \qquad G_M^+(Q^2) = G_M^p(Q^2) - G_M^n(Q^2)$$

$$K(E_c, Q^2) = -\int_{E_Q}^{\infty} \frac{dE}{E}[2g_1^{1/2}(x,Q^2) - g_1^{3/2}(x,Q^2)] - \frac{m_p}{8\pi^2\alpha_{em}} \int_{E_c}^{\infty} dE[2\Delta\sigma^{1/2} - \Delta\sigma^{3/2}]$$

$g_1^I, \Delta\sigma^I$: isovector photon + proton $\longrightarrow$ state with isospin I

(5)DGS representation for the current product

$$
\begin{aligned}
C_{ab}(p\cdot q, q^2) &= \int d^4x \exp(iqx) < p|[J_a(x), J_b(0)]|p >_c \\
&= \int d^4x \exp(iqx) \int_0^\infty d\lambda^2 \int_{-1}^1 d\beta h_{ab}(\lambda^2,\beta) i\Delta(x,\lambda^2) \\
&= (2\pi)\int_0^\infty d\lambda^2 \int_{-1}^1 d\beta \delta((q+\beta p)^2 - \lambda^2)\epsilon(q^0+\beta p^0) h_{ab}(\lambda^2,\beta)
\end{aligned}
$$

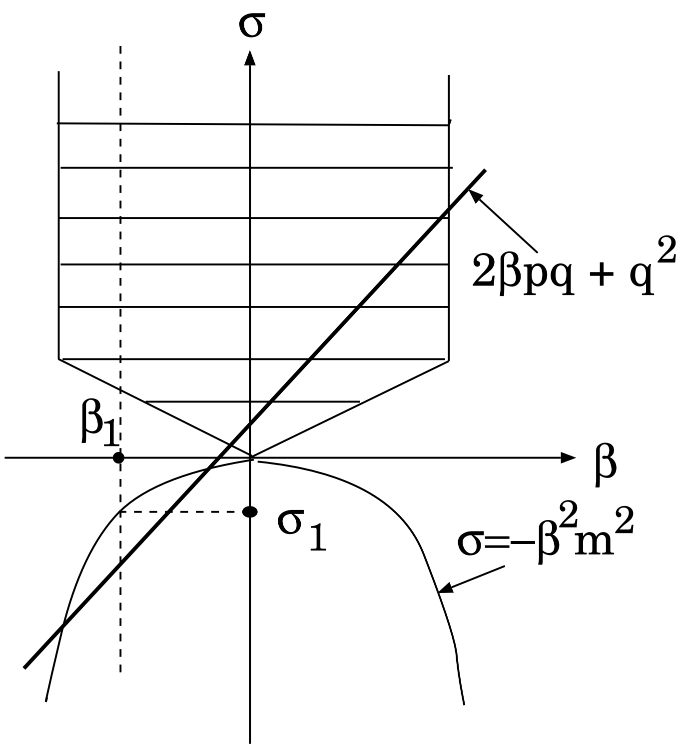
$$
\begin{aligned}
C_{ab}(p\cdot q, q^2) &= \sum_n (2\pi)^4 \delta^4(p+q-n)\langle p|J_a(0)|n\rangle\langle n|J_b(0)|p\rangle \\
&- \sum_n (2\pi)^4 \delta^4(p-q-n)\langle p|J_b(0)|n\rangle\langle n|J_a(0)|p\rangle
\end{aligned}
$$

take the rest frame: $p=(m,\vec{0})$

the first term: $m+q^0=n^0 \;\therefore\; q^0 \geqq M_s - m$

the second term: $m-q^0=n^0 \;\therefore\; q^0 \leqq m - M_u$

$m \leqq (M_s+M_u)/2 \longrightarrow$ the first term and the second term are disconnected.



$\sigma = \lambda^2 - \beta^2 m^2.$

Integration path: $\sigma = 2\beta p\cdot q + q^2$

at the rest frame $p\cdot q + \beta m^2 = m(q^0 + \beta p^0)$

$\therefore \epsilon(q^0 + \beta p^0) = \epsilon(p\cdot q + \beta m^2)$

$p\cdot q + \beta_1 m^2 = 0$,$\sigma_1 = -\beta_1^2 m^2$,

$\sigma_1 \geqq 2\beta_1 p\cdot q + q^2$

the sign change always in the causality forbidden region.

s channel $\rightarrow p\cdot q > 0 \rightarrow$ the slope is positive

only the region $\epsilon(p\cdot q + \beta m^2) = 1$ contributes.

$\therefore$ s channel and u channel are disconnected.

$$(2\pi)\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\delta((q+\beta p)^2-\lambda^2)h_{ab}(\lambda^2,\beta)\theta(q^0+\beta p^0)$$
$$=\sum_n(2\pi)^4\delta^4(p+q-n)\langle p|J_a(0)|n\rangle\langle n|J_b(0)|p\rangle$$

$$(2\pi)\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\delta((q+\beta p)^2-\lambda^2)h_{ab}(\lambda^2,\beta)\theta(-(q^0+\beta p^0))$$
$$=\sum_n(2\pi)^4\delta^4(p-q-n)\langle p|J_b(0)|n\rangle\langle n|J_a(0)|p\rangle$$

$$\begin{aligned}
W_{ab}(p\cdot q,q^2) &= \int d^4x\exp(iqx)<p|\{J_a(x),J_b(0)\}|p>_c\\
&= \int d^4x\exp(iqx)\int_0^\infty d\lambda^2\int_{-1}^1 d\beta h_{ab}(\lambda^2,\beta)i\Delta^{(1)}(x,\lambda^2)\\
&= (2\pi)\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\delta((q+\beta p)^2-\lambda^2)h_{ab}(\lambda^2,\beta)
\end{aligned}$$

(+ i) component of the anti-commutation relation on the null-plane

$$< p,s|\{J_a^+(x), J_b^i(0)\}|p,s>_c \delta(x^+)|_{spin}$$

$$= -\epsilon^{+i}{}_{\alpha\beta} < p,s|\partial^\alpha[\Delta^{(1)}(x) G_c^{5\beta}(x|0)]|p,s>_c$$

$$\Delta^{(1)}(x)|_{x^+=0} = -\frac{1}{2\pi}\ln|x^-|\delta(\vec{x}^\perp)$$

$$< p,s|G_c^{5\beta}(x|0)|p,s>_c = < p,s|d_{abc} S_c^{5\beta}(x|0) - f_{abc} A_c^{5\beta}(x|0)|p,s>_c$$

$$\begin{aligned} < p,s|S_c^{5\beta}(x|0)|p,s>_c &= s^\mu S_c^5(p\cdot x, x^2) + p^\mu (x\cdot s)\bar{S}_c^5(p\cdot x, x^2) \\ &+ x^\mu (x\cdot s)\tilde{S}_c^5(p\cdot x, x^2) \end{aligned}$$

(6)The sum rule from the (+ i) component

$$\int_0^1 \frac{dx}{x} g_1^{ab}(x,Q^2) = -\frac{1}{8\pi} d_{abc} \int_{-\infty}^{\infty} d\alpha \ln|\alpha| \{S_c^5(\alpha,0) + \alpha \bar{S}_c^5(\alpha,0))\}$$

$$\int_0^1 \frac{dx}{x} g_1^p(x,Q^2) = \int_0^1 \frac{dx}{x} g_1^p(x,Q_0^2)$$

$$G_1^p(\nu,0) = -\frac{1}{8\pi^2 \alpha_{em}} \{\sigma_{3/2}^{\gamma p}(\nu) - \sigma_{1/2}^{\gamma p}(\nu)\}$$

By separating out the Born term, we obtain

$$\int_{x_c}^{1} \frac{dx}{x} g_1^p(x,Q^2) = B(Q^2) - \frac{1}{8\pi^2\alpha_{em}} \int_{\nu_0}^{\nu_c} d\nu \{\sigma_{3/2}^{\gamma p} - \sigma_{1/2}^{\gamma p}\} + K(E_c,Q^2)$$

$$B(Q^2) = \frac{1}{2}\{F_1^p(0)(F_1^p(0) + F_2^p(0)) - F_1^p(Q^2)(F_1^p(Q^2) + F_2^p(Q^2))\}$$

$$K(E_c,Q^2) = \frac{1}{8\pi^2\alpha_{em}} \int_{\nu_c}^{\infty} d\nu \{\sigma_{1/2}^{\gamma p} - \sigma_{3/2}^{\gamma p}\} - \int_{\nu_c^Q}^{\infty} \frac{d\nu}{\nu} g_1^p(x,Q^2)$$

$$\nu_c = m_p E_c \quad \nu_c^Q = m_p E_Q \quad E_Q = E_c + Q^2/2m_p \quad x_c = \frac{Q^2}{2\nu_c^Q}$$

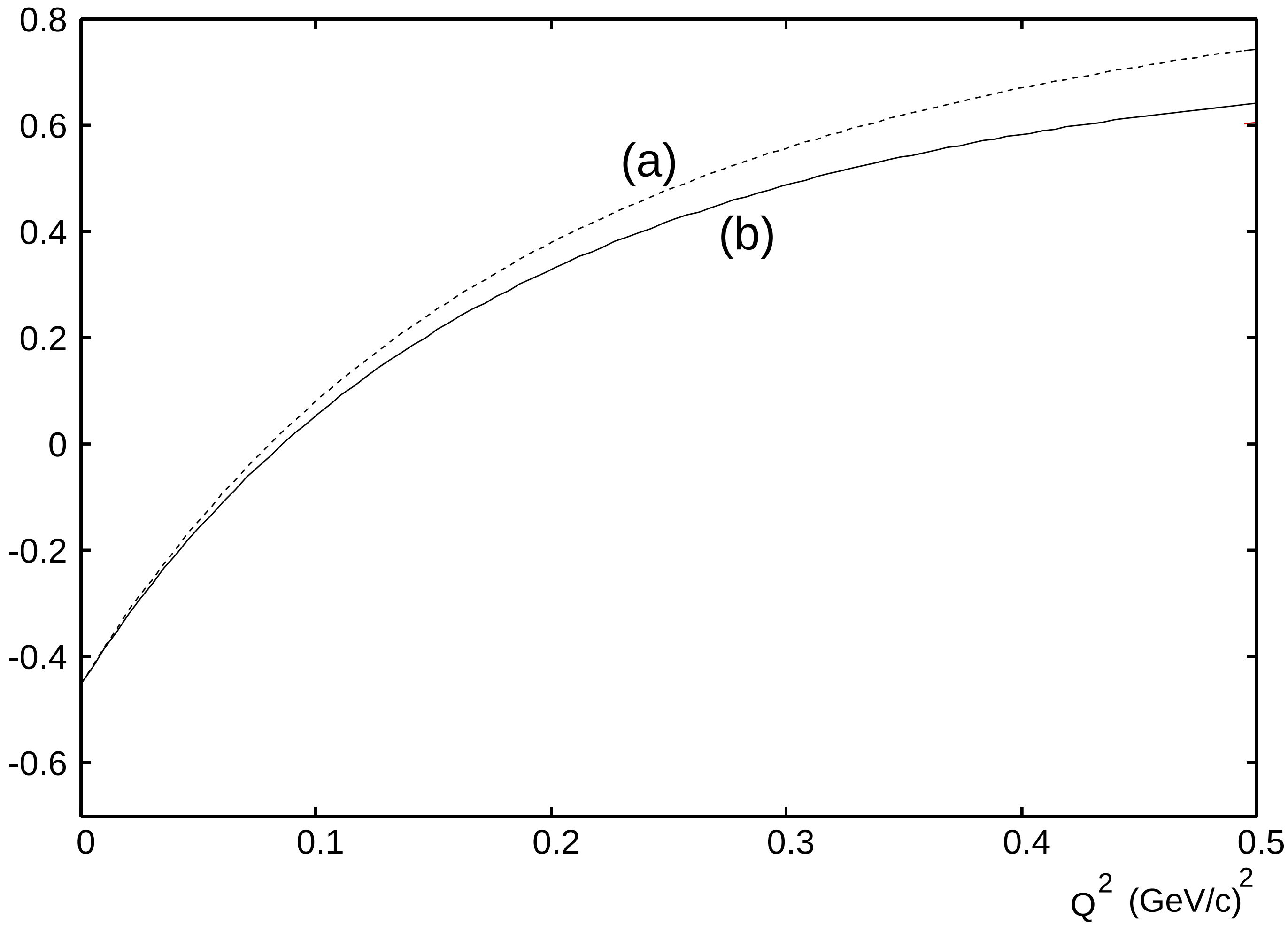
Using parameters in "S.Simula, M.Osipenko, G.Ricco and M.Taiuti, Phys.Rev. **D65**, 034017(2002)", $K(E_c,Q^2)$ is estimated as $K(2,0.05) \sim -0.015$ and $K(2,0.1) \sim -0.029$.

From the experimental data in “Dutz H, et al.(GDH Collaboration). Phys.Rev.Lett. **91**, 192001(2003)“, we find

$$\frac{m_N}{8\pi^2\alpha_{em}}\int_{E_0}^{2} dE\{\sigma^{\gamma p}_{3/2}-\sigma^{\gamma p}_{1/2}\}\sim 0.45.$$

We use the standard dipole fit

$$G^p_E=\frac{1}{(1+\frac{Q^2}{0.71})^2}\quad G^p_M=\mu_p G^p_E\quad G^p_E=F^p_1-\frac{Q^2}{4m_N^2}F^p_2\quad G^p_M=F^p_1+F^p_2$$



The estimate of the $\int_{x_c}^{1} \frac{dx}{x} g_1^p(x, Q^2)$ through the sum rule. The dotted curve (a) is the one given by neglecting the $K(2, Q^2)$ and the curve (b) is the one given by including the contribution from $K(2, Q^2)$.

(7)Conclusion

We find, in the small Q^2 region near $Q^2 \sim 0.1$(GeV/c)^2, that the integral $\displaystyle\int_{x_c}^{1} \frac{dx}{x} g_1^p(x, Q^2)$ becomes zero and that it changes the sign from the negative to the positive. This behavior is caused by the rapid change of the resonances to compensate the rapid change of the elastic to satisfy the sum rule. It is this rapid change of the resonances which gives the sign change of the GDH sum. Hence we see why it occurs in the very small Q^2 region.