Nucleon-Lambda interaction from lattice QCD

土居孝寛 (Takahiro Doi in Kyoto Univ.)

And HAL QCD collaboration.

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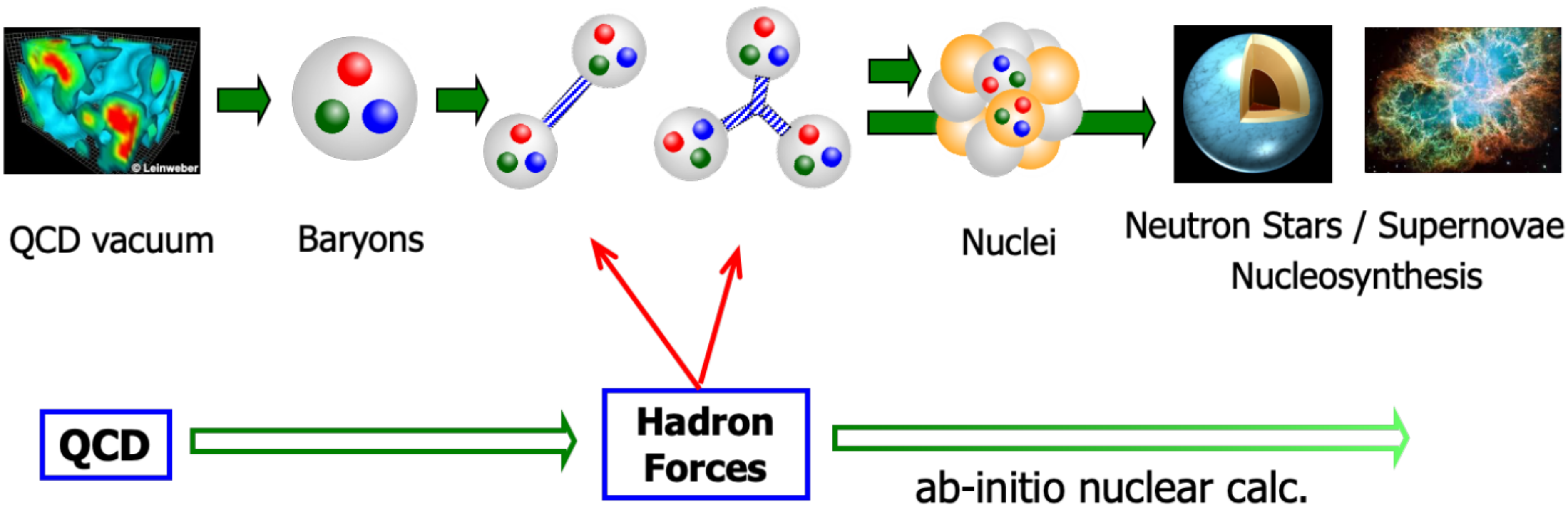
Hadrons to Atomic nuclei HAL from Lattice QCD

2023 05/17 BISHOP 2023, online talk

Purpose of HAL QCD collaboration:

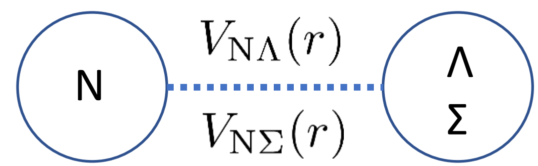
To obtain the hadron-hadron interaction from the first-principles calculation of QCD.



Our hadron-hadron interaction can be input of many-body calculation of hadrons, then we want to quantitatively understand phenomena related to hadron physics.

Baryon-Baryon interactions in Strangeness=-1

☞ S=-1: NΛ-NΣ potentials



◎importance

- They are important to go from nuclear physics (including only nucleons), to strangeness nuclear physics(nucleons + hyperons).

- Experiment for NΛ-NΣ is more difficult than experiment of NN. Then, it is important to determine the interaction by theoretical calculations(lattice QCD).

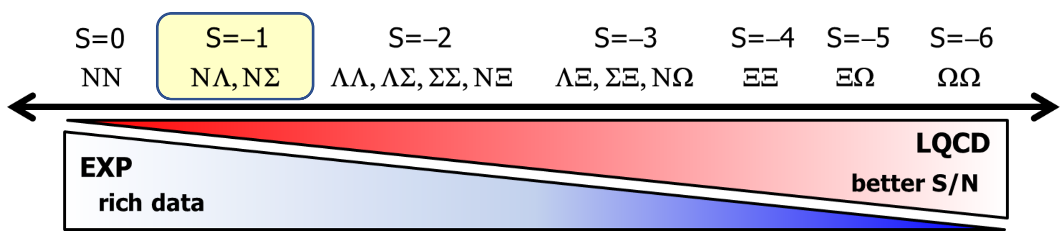
- NΛ-NΣ interaction can be determined also by recent experiments at J-PARC, and HAL QCD potential can be directly compared to the experimental results.

◎Application

- Spectroscopy of hyper nucleus
- Microscopic understanding of Inner structure of a neutron star.

◎Difficult

- large error (light baryons)
- Bad signals due to contamination from higher excited states ← discussed later



Outline

- **Generation of Gauge Configuration on Supercomputer Fugaku(Only results)**
- NΛ-NΣ potential
- Outlook

nearly physical point

physical point

K-conf.

Nf=2+1, Iwasaki gauge + clover fermion action

beta=1.82 ($1/a \simeq 2.3$ GeV)

$96^4 \leftrightarrow (8.1\mathrm{fm})^4$

$(\kappa_{u,d}, \kappa_s) = (0.126117, 0.124790)$

$m_\pi \simeq 146$MeV, $m_K \simeq 525$ MeV

F-conf.

Nf=2+1, Iwasaki gauge + clover fermion action

beta=1.82 ($1/a \simeq 2.3$ GeV)

$96^4 \leftrightarrow (8.1\mathrm{fm})^4$

$(\kappa_{u,d}, \kappa_s) = (0.126117, 0.124902)$

total independent conf=1600conf.

(320 conf. x 5 run = 1600 conf.)

Preliminary results of hadron mass

π 137 [MeV]

K 502 [MeV]

N 940 [MeV]

Experimental data(Particle Data Group 2020)

$m_{\pi^+} \simeq 139.57$MeV, $m_{\pi^0} \simeq 134.98$MeV

Isospin averaged pion mass $m_\pi \simeq 138.0$MeV

$m_{K^+} \simeq 493.68$MeV, $m_{K^0} \simeq 497.61$MeV

Isospin averaged Kaon mass $m_K \simeq 495.6$MeV

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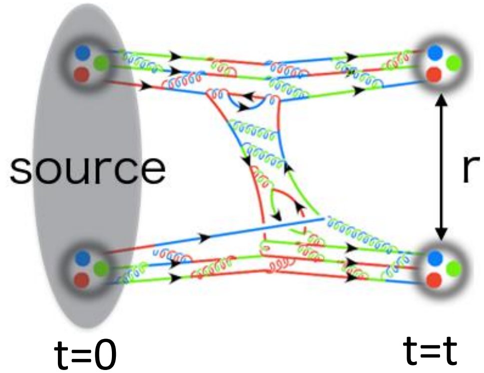
- NΛ & NΣ I=1/2
  - 1S0 central NΛ-NΣ potential
  - 3S1-3D1 central & tensor NΛ-NΣ potential
- NΣ I=3/2
  - 1S0 central NΣ potential
  - 3S1-3D1 central & tensor NΣ potential

HAL QCD method

Ishii, Aoki & Hatsuda, Phys. Rev. Lett. 99 (2007) 022001
Ishii+ [HAL QCD Coll.], Phys. Lett. B712 (2012) 437

In the case of NN potential

$$G_{NN}(\boldsymbol{r},t) = \langle 0|N(\boldsymbol{r},t)N(\boldsymbol{0},t)|\overline{J_{\rm src}(t=0)}|0\rangle$$



t: imaginary time on lattice

Nambu-Bethe-Salpeter(NBS) wave function with relative momentum k is obtained at infinite t

$$\underset{t\to\infty}{G_{NN}(\boldsymbol{r},t) \to} \psi_{l,k}(\boldsymbol{r}) \simeq A_{l,k}\frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} \quad (r > R)$$

R: interaction range

NBS wave function is a solution of Schrödinger eq. with **NN potential**.

- We can extract **scattering phase shift** from NBS wave function.
- **NN potential** can be calculated so that Schrödinger eq. has NBS w.f. as solution.

(time-dependent) HAL QCD method

Ishii+ [HAL QCD Coll.], Phys. Lett. B712 (2012) 437

In the case of NN potential

$$G_{NN}(\boldsymbol{r},t) = \langle 0|N(\boldsymbol{r},t)N(\boldsymbol{0},t)|\overline{J_{\mathrm{src}}(t=0)}|0\rangle$$

$$R(\boldsymbol{r},t) \equiv G_{NN}(\boldsymbol{r},t)/G_N(t)^2$$

$$= \sum_i A_{W_i}\psi_{W_i}(\boldsymbol{r})\mathrm{e}^{-(W_i-2m)t}$$

Many states contributes

i: each energy eigen state

Under inelastic threshold, all excited scattering states share the same U(r,r'):

$$(\nabla^2 + k_{W_i})\psi_{W_i}(\boldsymbol{r}) = m\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi_{W_i}(\boldsymbol{r}')$$

- All equations(i=0,1,2,3,… up to elastic threshold) can be combined as

$$(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} + \frac{\nabla^2}{m})R(\boldsymbol{r},t) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t)$$

- Local potential is obtained by derivative expansion

$$U(\boldsymbol{r},\boldsymbol{r}') = V_{\mathrm{C}}(r) + V_{\mathrm{T}}(r)S_{12} + V_{\mathrm{LS}}(r)\boldsymbol{L}\cdot\boldsymbol{S} + \cdots$$

LO LO NLO

Partial wave(L=0,2) decomposition on the lattice

Method 1. A_1^+ projection of cubic group

M. Luscher, Nucl. Phys. B 354 (1991), 531.
Aoki, Hatsuda, Ishii, PTEP 123 (2010).

$$R^{A_1^+}(\boldsymbol{r}) \equiv \frac{1}{48}\sum_{g\in O_h} R(g^{-1}\boldsymbol{r})$$

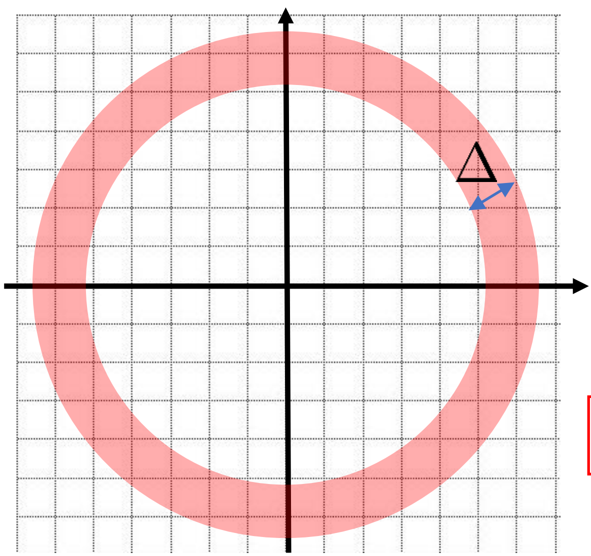
: This has dominant contribution from L=0 and small contribution from L=4,6,….

S-wave $R_{\mathrm{S}}(\boldsymbol{r}) = R^{A_1^+}(\boldsymbol{r})$

D-wave $R_{\mathrm{D}}(\boldsymbol{r}) = R(\boldsymbol{r}) - R^{A_1^+}(\boldsymbol{r})$

Method 2. Misner's method

C. W. Misner, Class. Quant. Grav. 21 (2004) S243.
T. Miyamoto et al., Phys. Rev. D 101 (2020) 074514.



Use $$R(\boldsymbol{r}) = \sum_{n,l,m} c^{\Delta}_{nlm} G^{\Delta}_{n}(r) Y_{lm}(\theta,\phi)$$

new basis function in r(radial direction)

instead of $$R(\boldsymbol{r}) = \sum_{l,m} g_{lm}(r) Y_{lm}(\theta,\phi)$$

sophisticated partial wave decomposition on the lattice

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S=-1

- NΛ & NΣ I=1/2
  - 1S0 central NΛ-NΣ potential — signal: bad
  - 3S1-3D1 central & tensor NΛ-NΣ potential — signal: good
- NΣ I=3/2
  - **1S0 central NΣ potential** — signal: good
  - 3S1-3D1 central & tensor NΣ potential — signal: bad

$N\Sigma$ **potential**

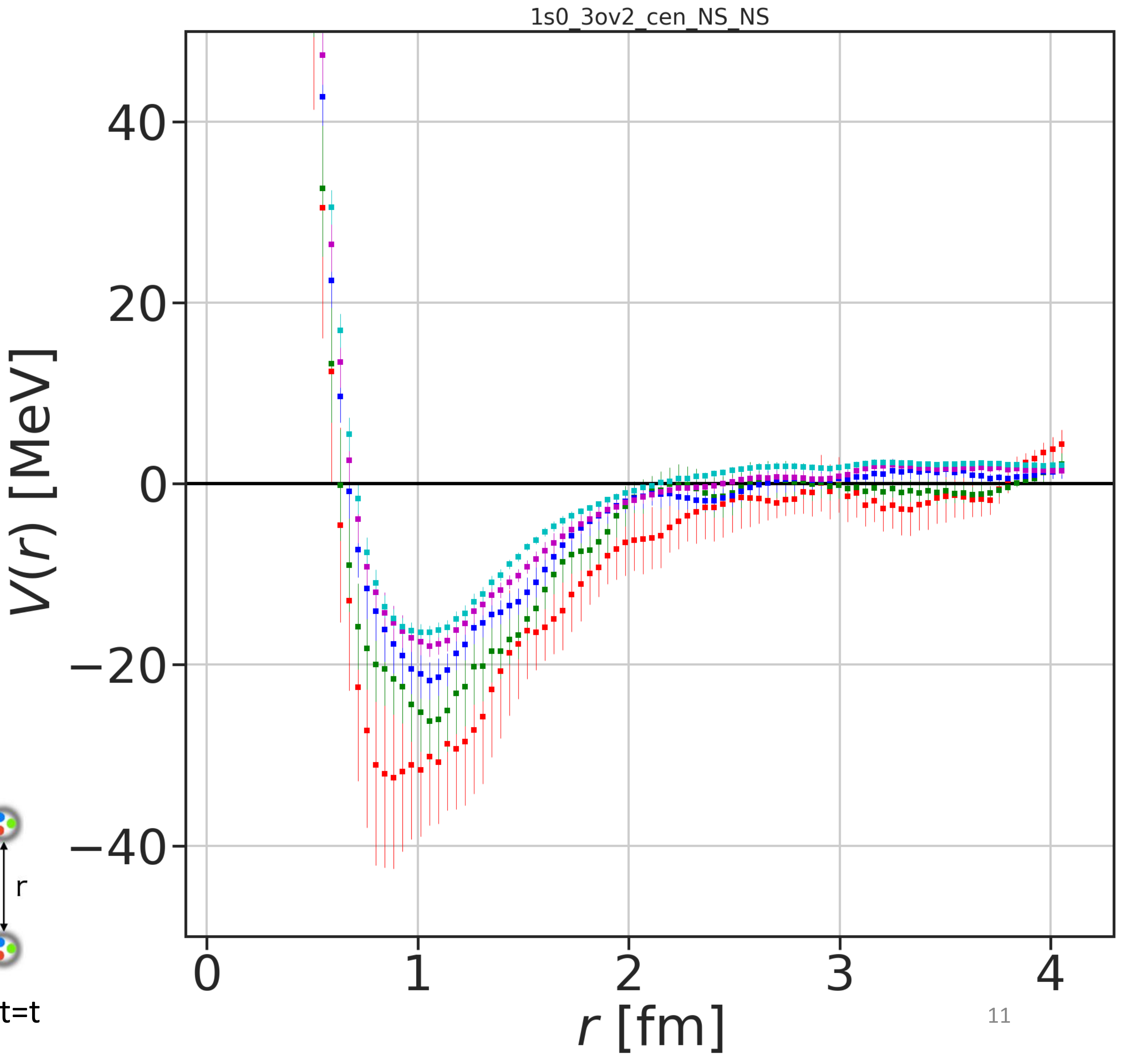
1S0, I=3/2

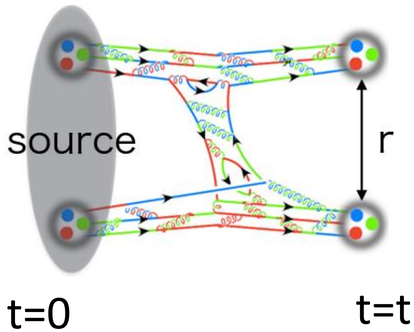
central

binsize=80

Nconf=800

w/ Misner





ΞΣ **potential**

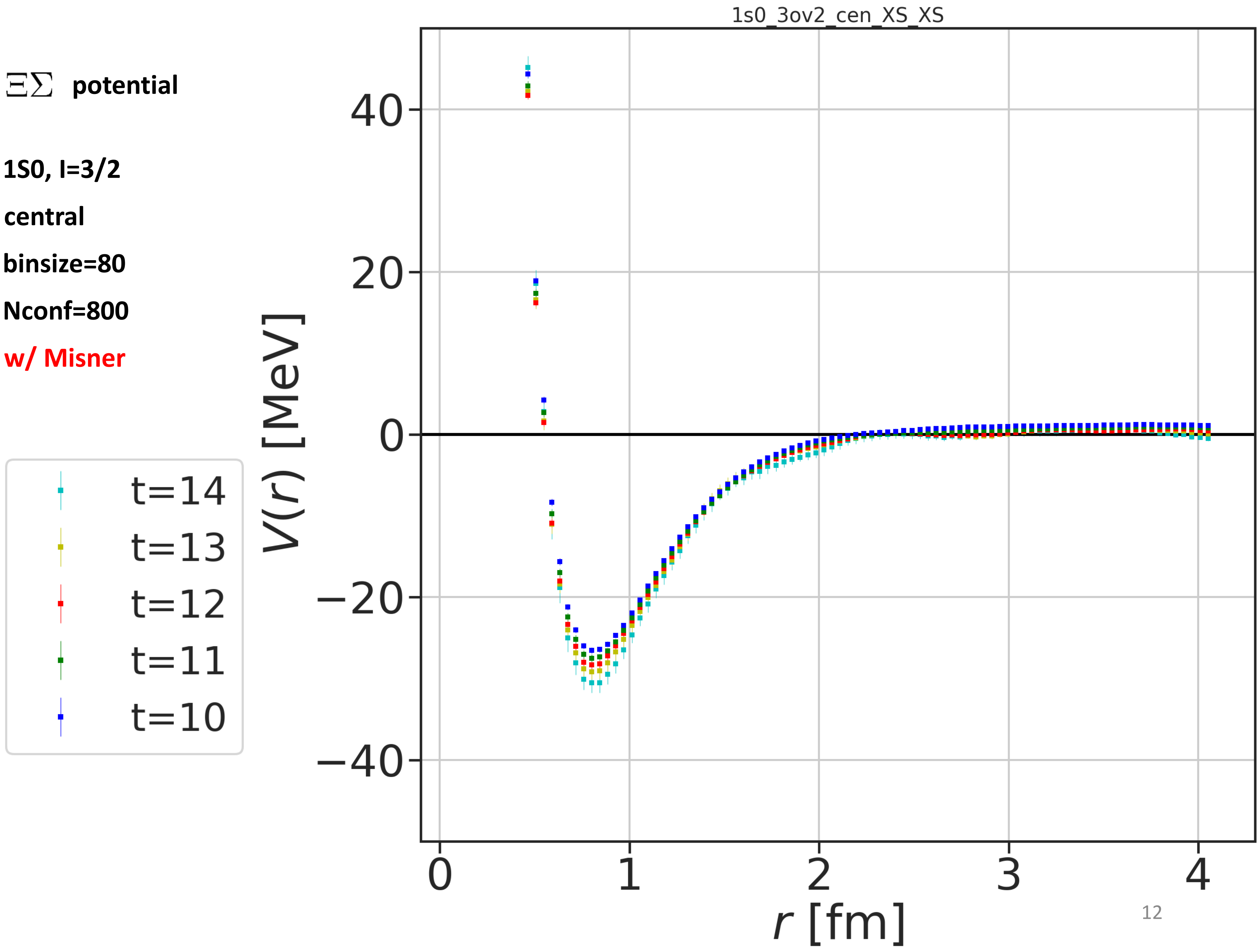
1S0, I=3/2

central

binsize=80

Nconf=800

w/ Misner



$\mathrm{N}\Sigma$ **potential**

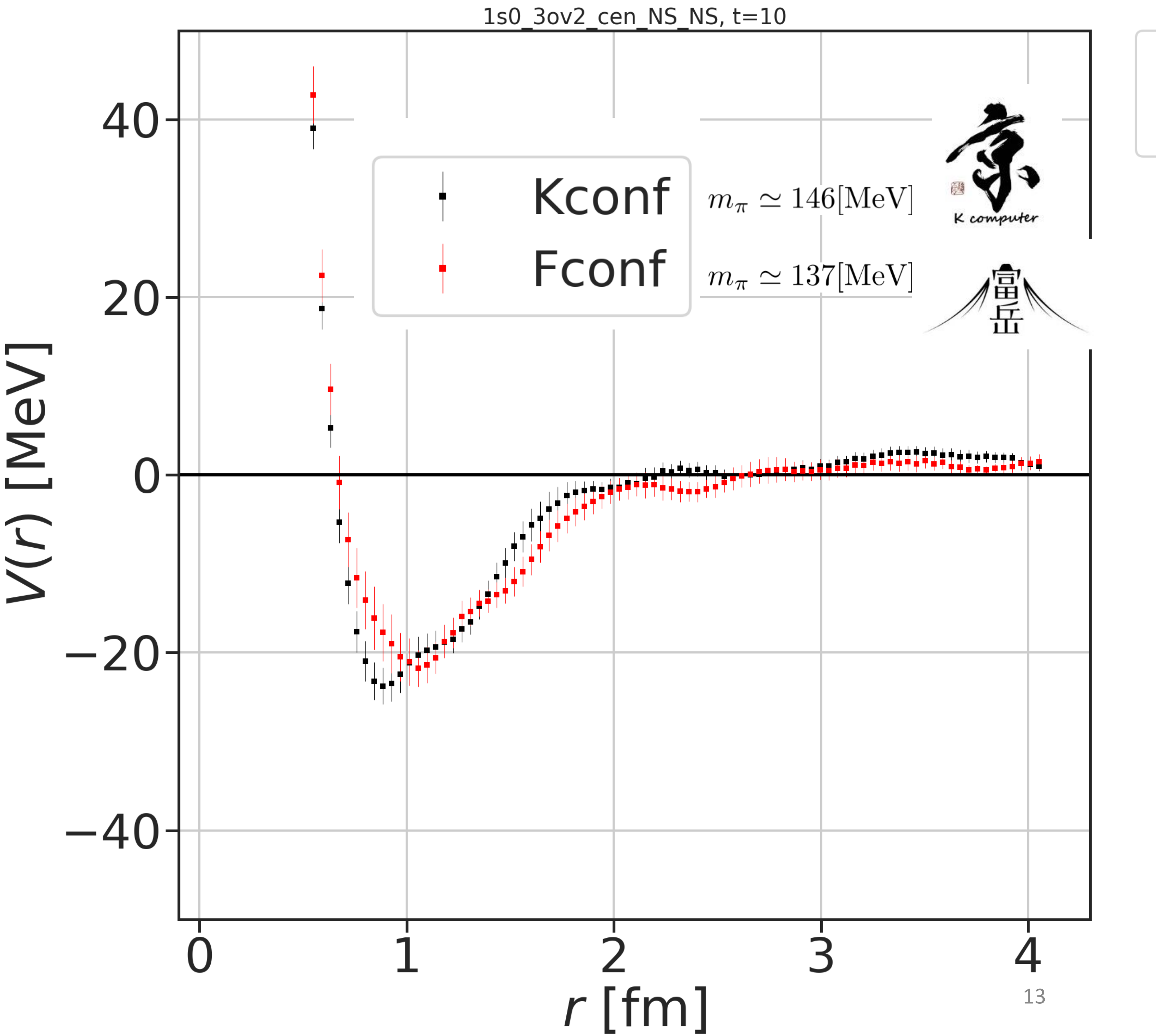
1S0, I=3/2

central

binsize=80

Nconf=800

w/ Misner



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		3S1-3D1 central & tensor NΛ-NΣ potential	good
	NΣ I=3/2	1S0 central NΣ potential	good
		3S1-3D1 central & tensor NΣ potential	bad

$N\Lambda - N\Sigma$

coupled channel potential

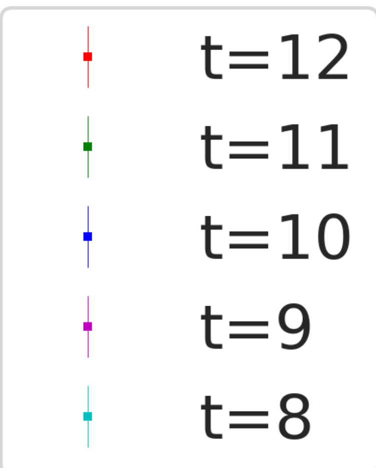
3S1, I=1/2

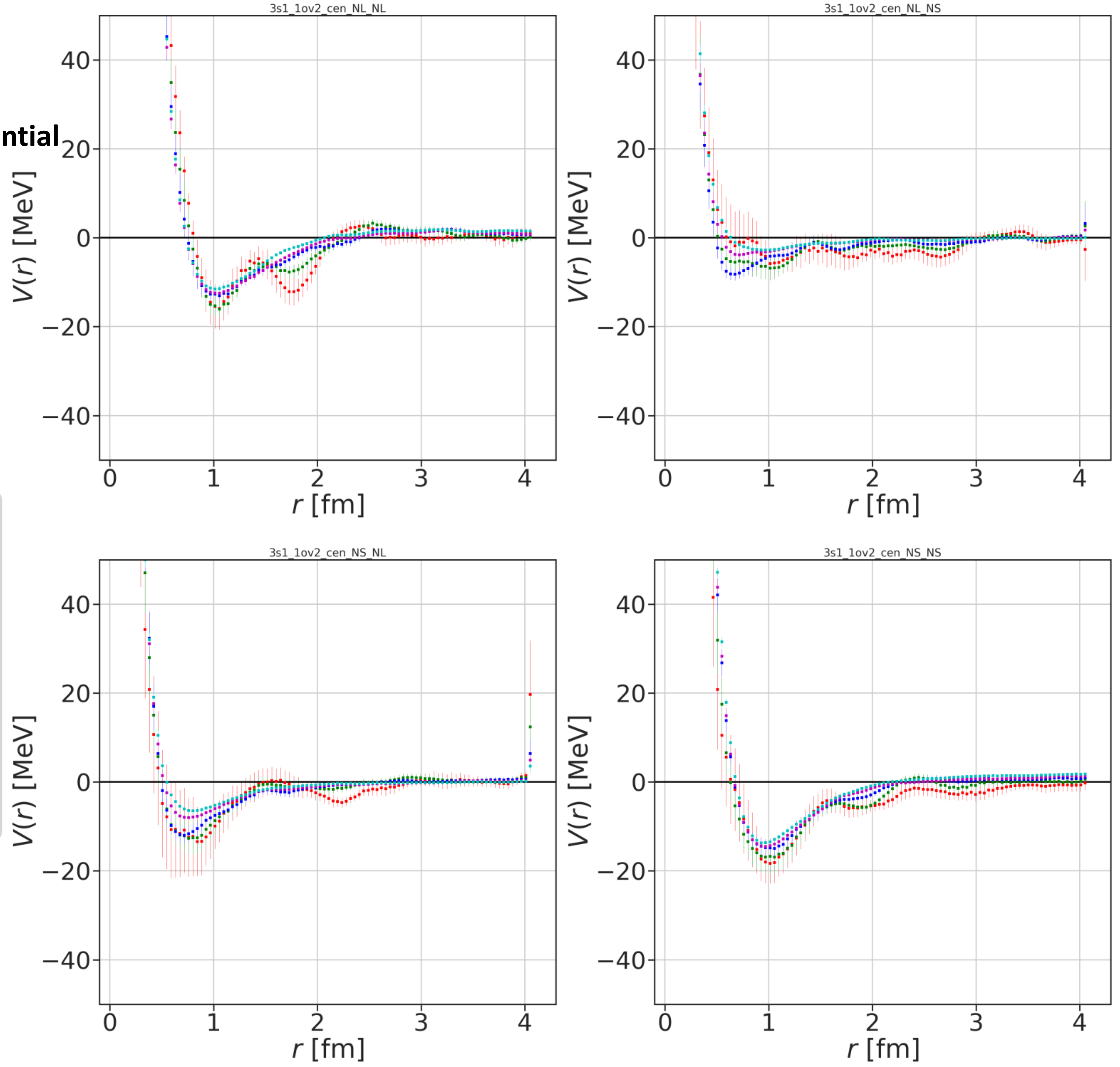
central

binsize=80

Nconf=800

w/ Misner





$N\Lambda - N\Sigma$

coupled channel potential

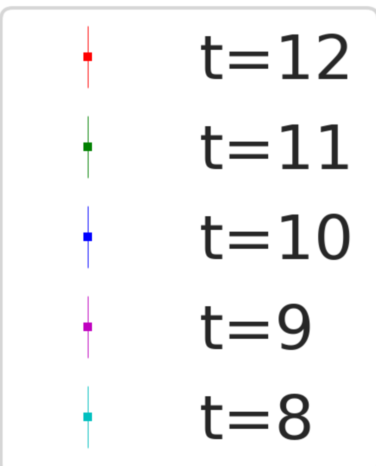
3S1, I=1/2

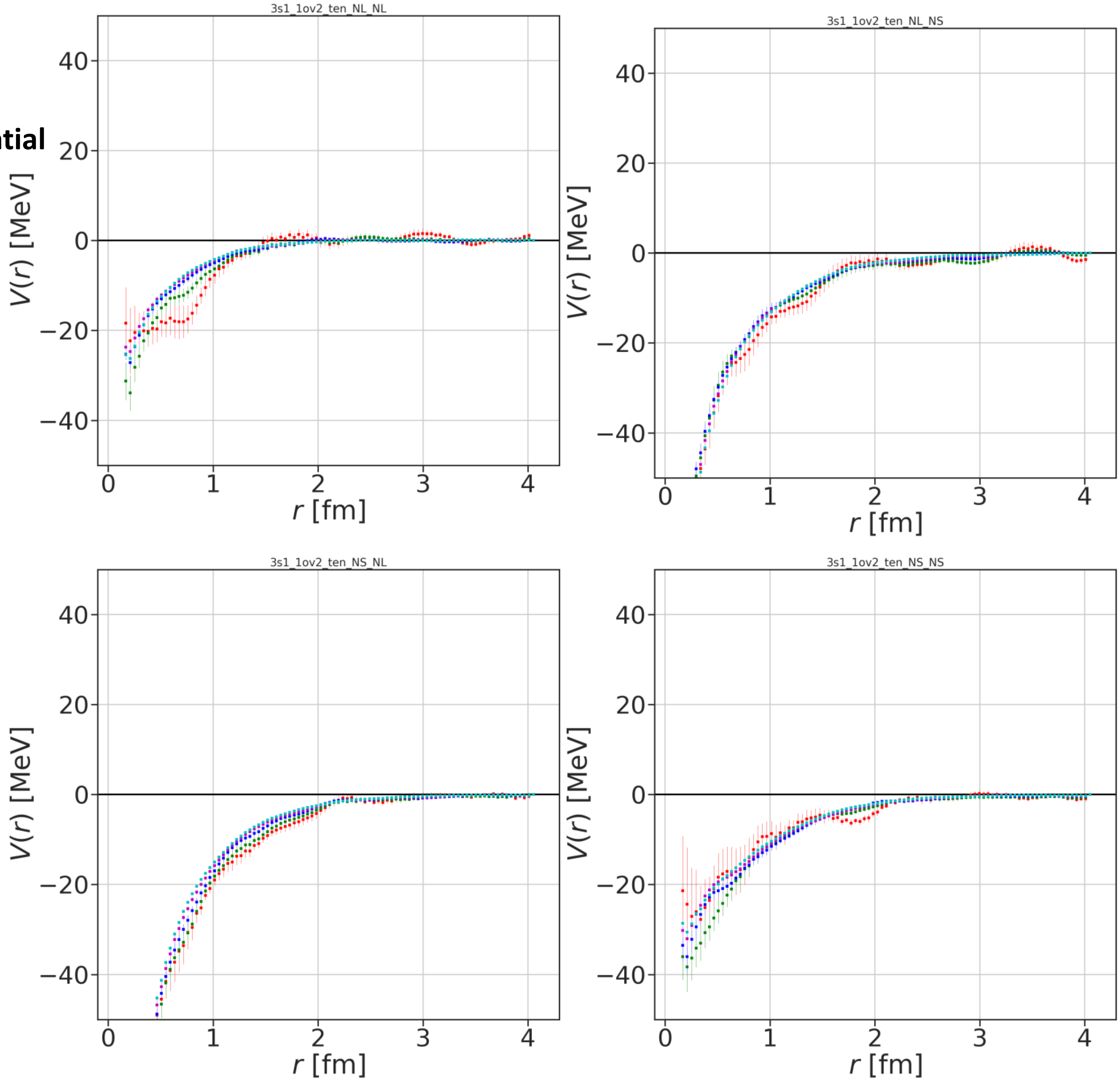
tensor

binsize=80

Nconf=800

w/ Misner





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	NΣ I=3/2	1S0 central NΣ potential	good
		3S1-3D1 central & tensor NΣ potential	bad

$N\Lambda - N\Sigma$

coupled channel potential

1S0, I=1/2

central

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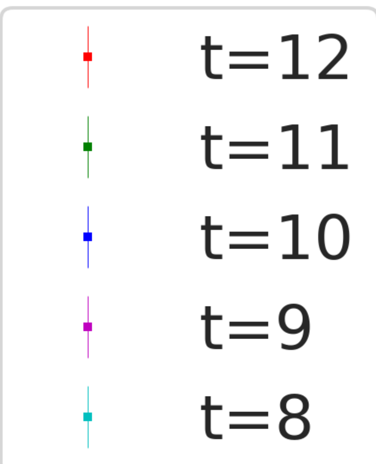
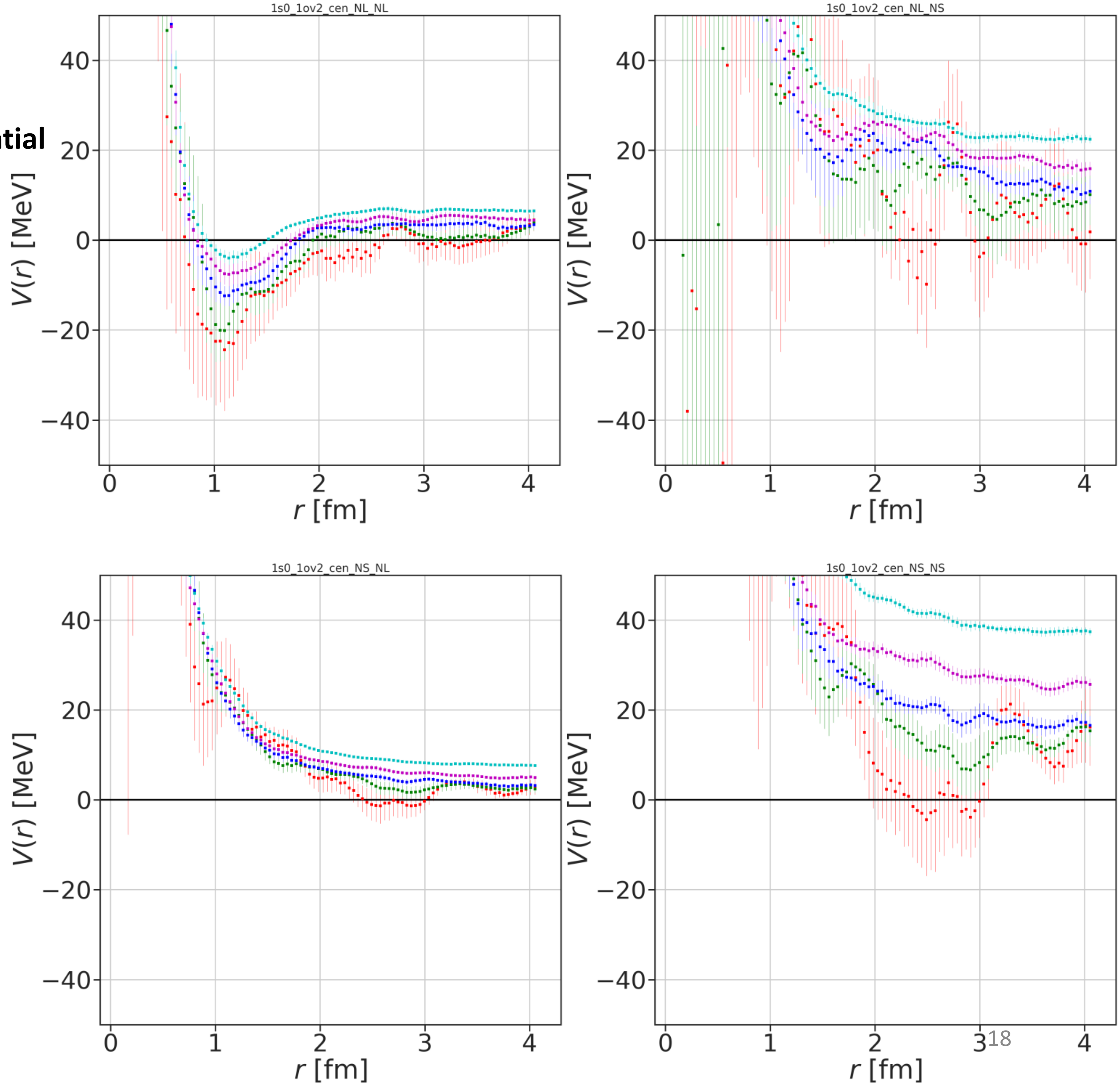
w/ Misner





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$N\Sigma$ **potential**

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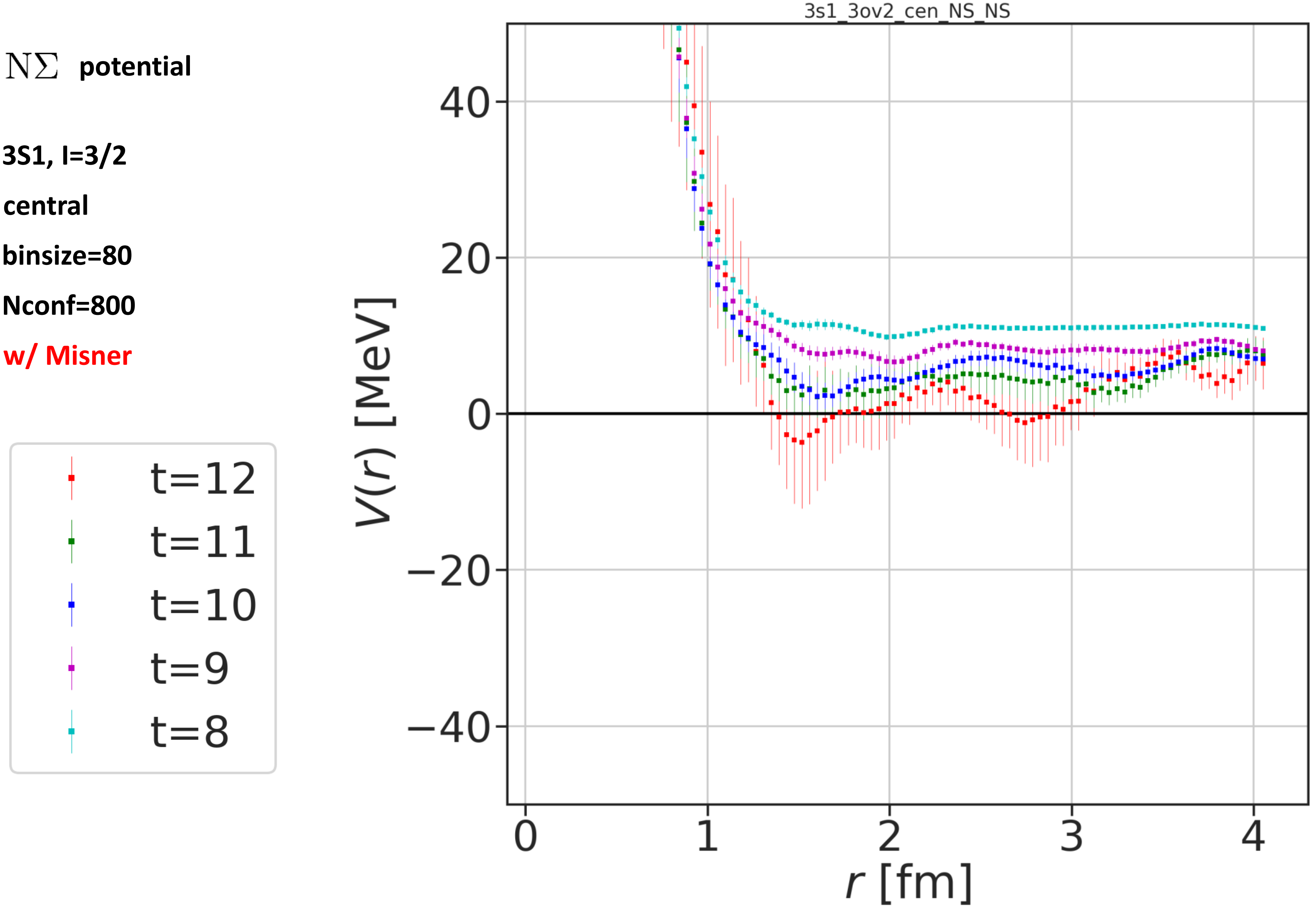

NΣ potential

3S1, I=3/2

tensor

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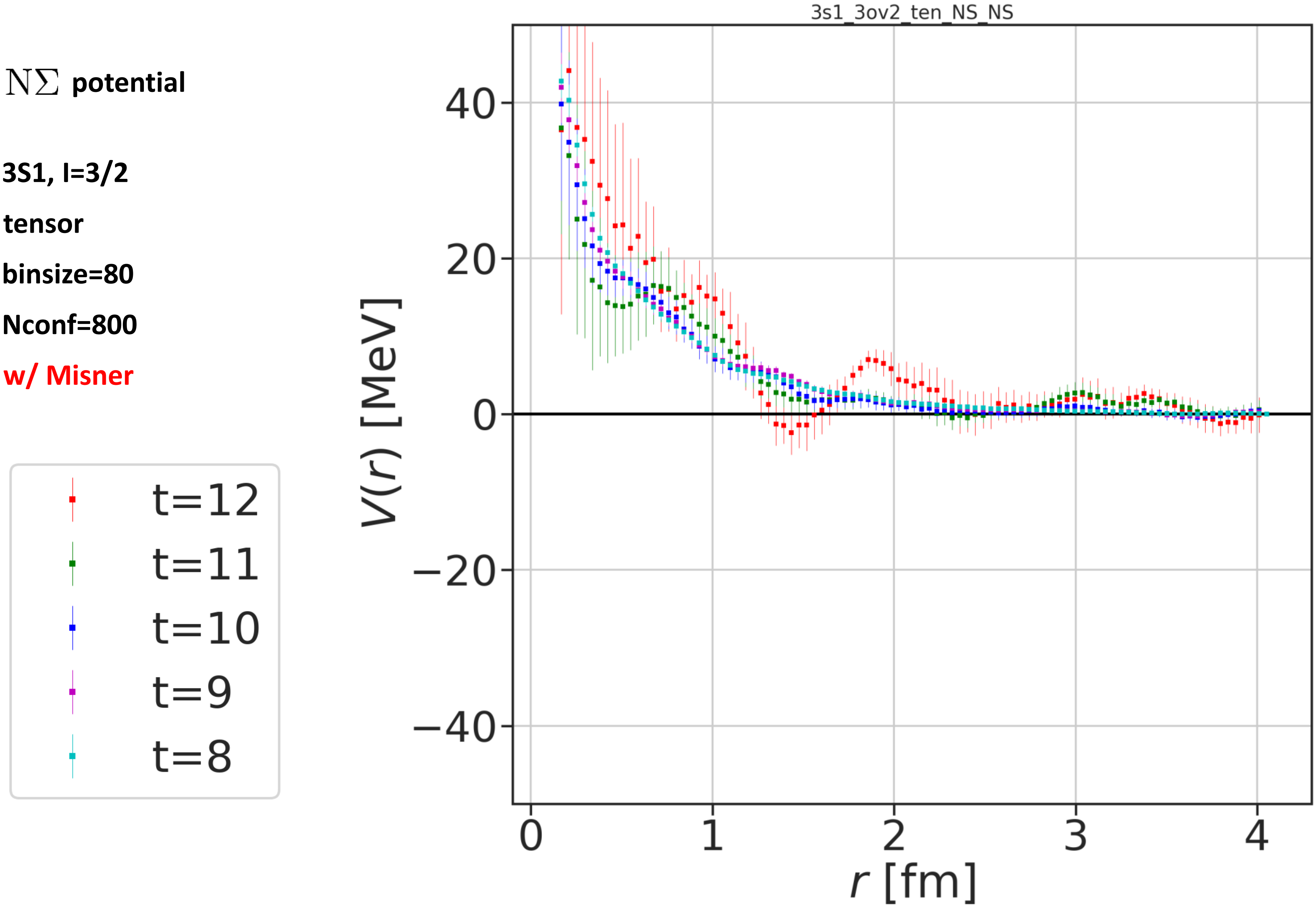
Nconf=800

w/ Misner



T. Inoue et al. [HAL QCD Collaboration], Prog. Theor. Phys. 124, 591 (2010).

attractive

	flavor multiplet	baryon pair (isospin)
spin 1S0	**27**	$\{NN\}$(I=1), $\{N\Sigma\}$(I=3/2), $\{\Sigma\Sigma\}$(I=2), $\{\Sigma\Xi\}$(I=3/2), $\{\Xi\Xi\}$(I=1)
	$\mathbf{8}_s$	none
	1	none
3S1	$\mathbf{10}^*$	$[NN]$(I=0), $[\Sigma\Xi]$(I=3/2)
	10	$[N\Sigma]$(I=3/2), $[\Xi\Xi]$(I=0)
	$\mathbf{8}_a$	$[N\Xi]$(I=0) repulsive

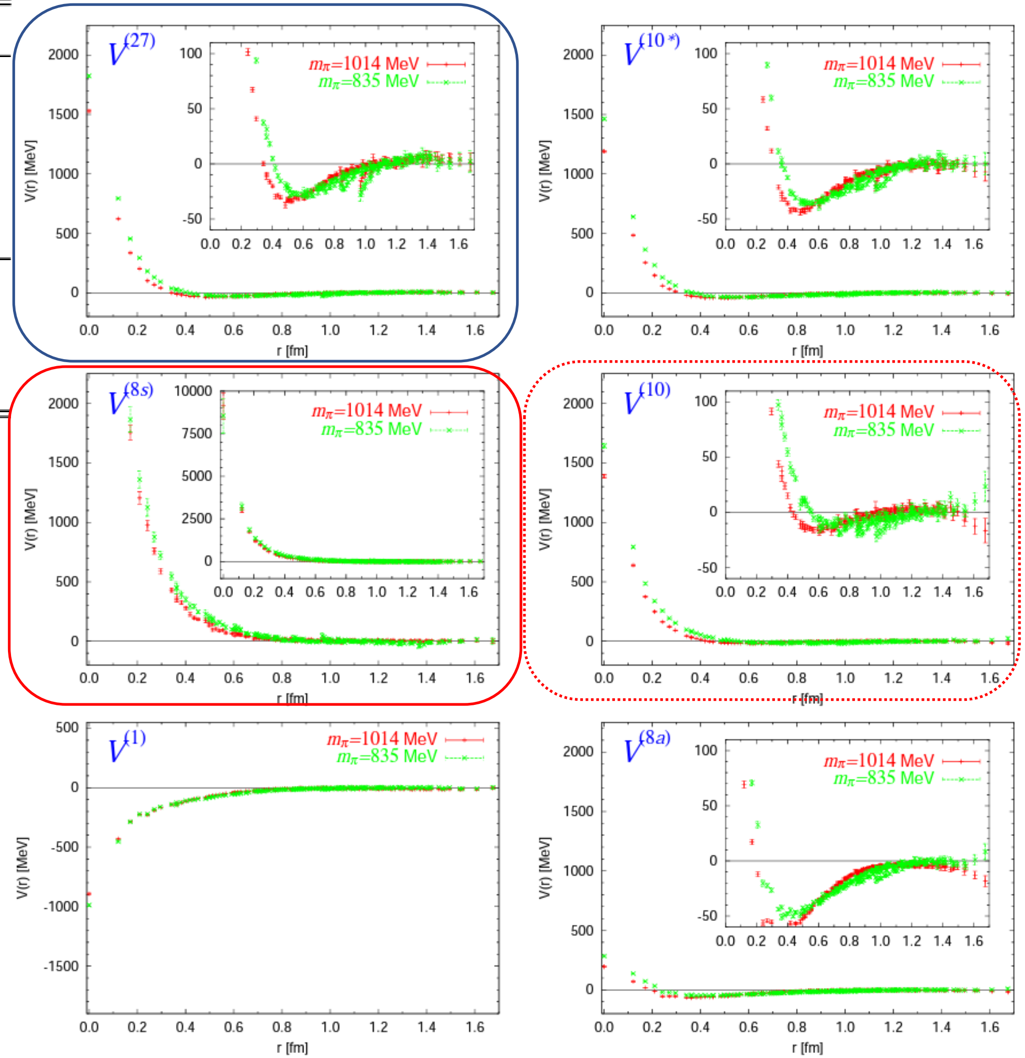
$S=-1,\ I=1/2,\ {}^1S_0$ sector.

repulsive

$$\begin{pmatrix} \langle N\Lambda| \\ \langle N\Sigma| \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{9}{10}} & -\sqrt{\frac{1}{10}} \\ \sqrt{\frac{1}{10}} & \sqrt{\frac{9}{10}} \end{pmatrix} \begin{pmatrix} \langle \mathbf{27}| \\ \langle \mathbf{8}_s| \end{pmatrix}$$

$S=-1,\ I=1/2,\ {}^3S_1$ sector.

$$\begin{pmatrix} \langle N\Lambda| \\ \langle N\Sigma| \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} \langle \mathbf{10}^*| \\ \langle \mathbf{8}_a| \end{pmatrix}$$



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  - 1S0 central NΛ-NΣ potential
  - 3S1-3D1 central & tensor NΛ-NΣ potential
- NΣ I=3/2
  - 1S0 central NΣ potential
  - 3S1-3D1 central & tensor NΣ potential

We want to extract signals

$$G_{\mathrm{N}\Lambda}(\mathbf{r},t) = \langle 0|N(\mathbf{r},t)\Lambda(\mathbf{0},t)|\overline{J_{\mathrm{src}}(t=0)}|0\rangle$$

$$R(\mathbf{r},t) \equiv \frac{G_{\mathrm{N}\Lambda}(\mathbf{r},t)}{G_{\mathrm{N}}(t)G_{\Lambda}(t)}$$

Many states contributes

$$= \sum_i A_{W_i}\psi_{W_i}(\mathbf{r})\mathrm{e}^{-(W_i - m_N - m_\Lambda)t}$$

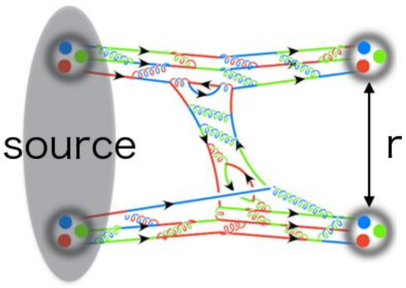
i: each energy eigen state

$$R(\mathbf{r},t) = R^{\mathrm{signal}}(\mathbf{r},t) + R^{\mathrm{inelastic}}(\mathbf{r},t) \qquad (R^{\mathrm{inelastic}}(\mathbf{r},t) \to 0 (t \to \infty))$$

We can get only LHS from lattice QCD, but we want to get only first term in RHS.
(Second term is noise from inelastic excited states)

If we take large t enough, second term will vanish, but this method does not work in practice
Then, we want to subtract second term other than taking large t enough.

Approximately subtract inelastic contamination



Consider inelastic contamination into one-baryon correlator:

$$G_{\mathrm{B}}(t) = \sum_{\mathbf{r}} \langle 0 | B(\mathbf{r}, t) | \overline{J_{\mathrm{src}}(t=0)} | 0 \rangle$$

$$G_{\mathrm{B}}^{\mathrm{ela}}(t) \equiv A_{\mathrm{B}} \mathrm{e}^{-m_{\mathrm{B}} t}$$ Fitted function

$$G_{\mathrm{B}}^{\mathrm{inela}}(t) \equiv G_{\mathrm{B}}(t) - G_{\mathrm{B}}^{\mathrm{ela}}(t)$$

Estimate the inelastic contamination of two-baryon correlator(NBS wave function) using the inelastic contamination of one-baryon correlator

$$G_{\mathrm{N}\Lambda}^{\mathrm{inela}}(t) = G_{\mathrm{N}}^{\mathrm{ela}}(t) G_{\Lambda}^{\mathrm{inela}}(t) + G_{\mathrm{N}}^{\mathrm{inela}}(t) G_{\Lambda}^{\mathrm{ela}}(t) + G_{\mathrm{N}}^{\mathrm{inela}}(t) G_{\Lambda}^{\mathrm{inela}}(t)$$

Nucleon 2pt corr. Lambda 2pt corr.

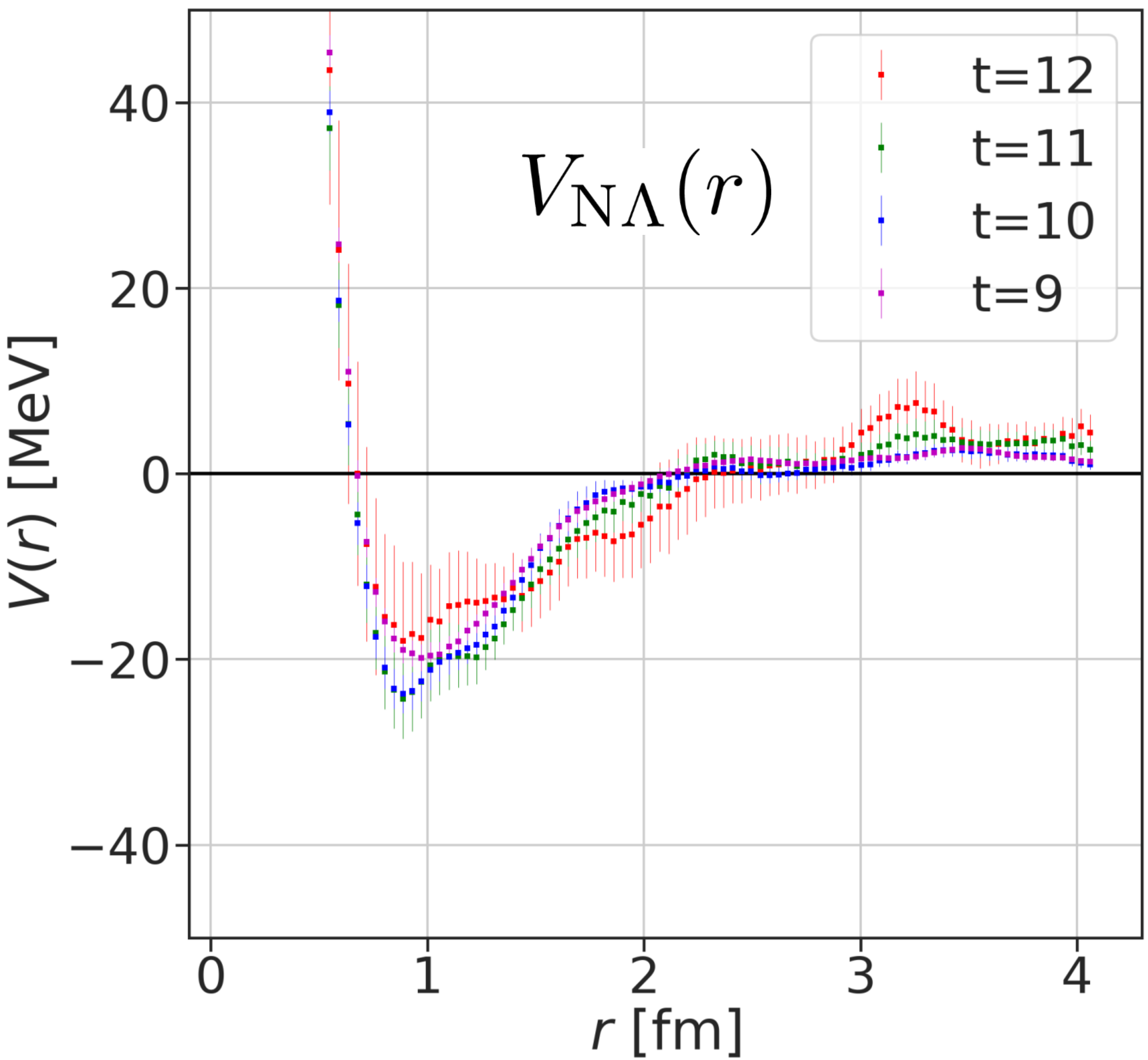
Calculate potentials using improved two-baryon correlator:

$$G_{\mathrm{N}\Lambda}(\mathbf{r}, t) \to G_{\mathrm{N}\Lambda}(\mathbf{r}, t) - \alpha G_{\mathrm{N}\Lambda}^{\mathrm{inela}}(t)$$

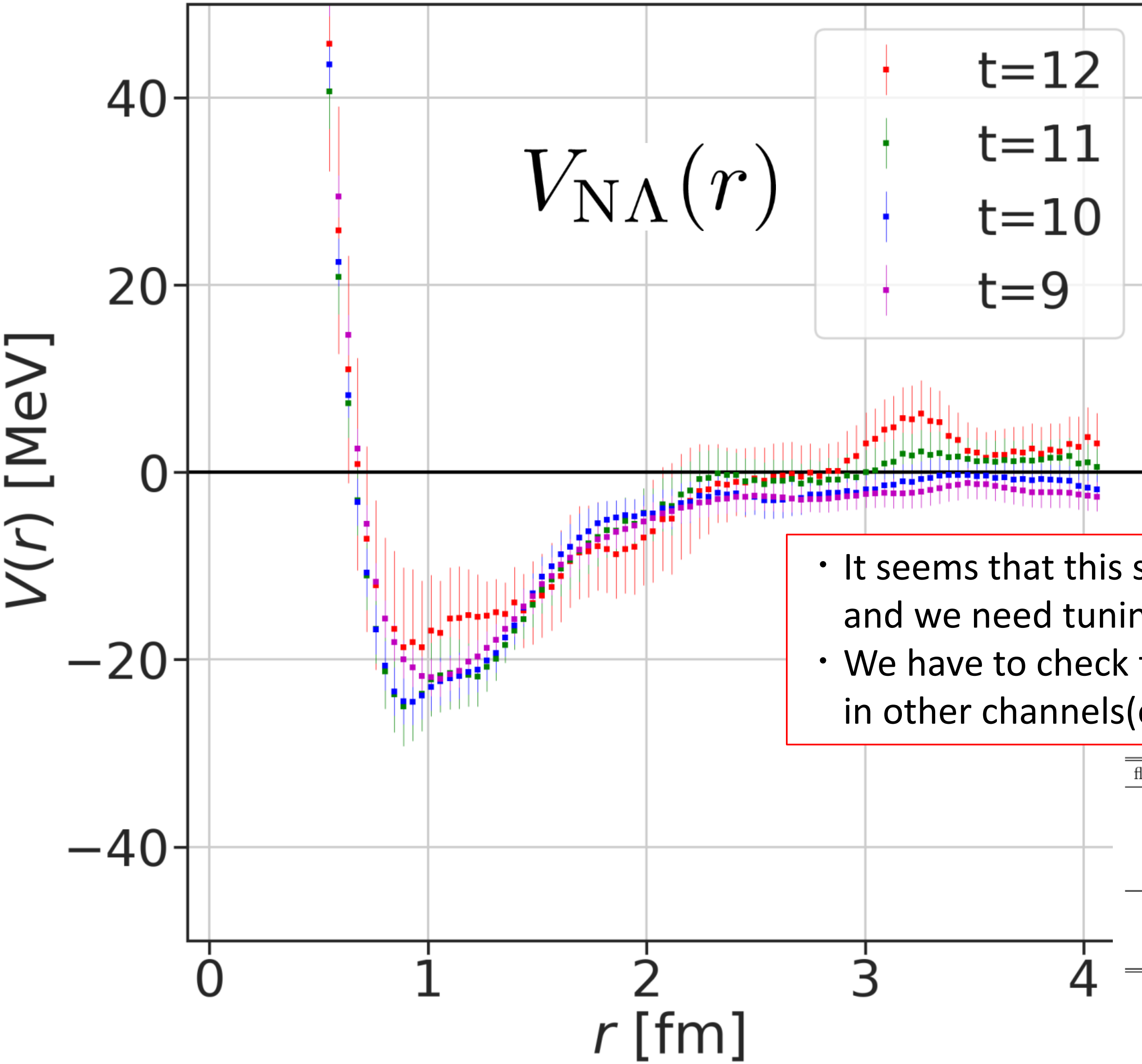
In the case of free gauge configuration: $G_{\mathrm{N}\Lambda}(\mathbf{r}, t) = \frac{1}{4L^3} G_{\mathrm{N}}(t) G_{\Lambda}(t)$ $\alpha = \frac{1}{4V}$

original results

$$R(\mathbf{r}, t) = \frac{G_{N\Lambda}(\mathbf{r}, t)}{G_N(t) G_\Lambda(t)}$$



Approximately subtract inelastic contamination



$$\tilde{R}(\mathbf{r},t) = \frac{G_{N\Lambda}(\mathbf{r},t) - \alpha G^{\text{inela}}_{N\Lambda}(t)}{G^{\text{ela}}_{N}(t) G^{\text{ela}}_{\Lambda}(t)} = R(\mathbf{r},t) - \alpha R^{\text{inela}}(t)$$

$$\alpha = \frac{1}{4V}$$

$$R(\mathbf{r},t) \equiv \frac{G_{N\Lambda}(\mathbf{r},t)}{G^{\text{ela}}_{N}(t) G^{\text{ela}}_{\Lambda}(t)}$$

- It seems that this subtraction works well and we need tuning of α for better results.
- We have to check that this subtraction works in other channels(e.g. ΞΞ)

flavor multiplet	baryon pair (isospin)
27	$\{NN\}$(I=1), $\{N\Sigma\}$(I=3/2), $\{\Sigma\Sigma\}$(I=2), $\{\Sigma\Xi\}$(I=3/2), $\{\Xi\Xi\}$(I=1)
$\mathbf{8}_s$	none
1	none
$\mathbf{10}^*$	$[NN]$(I=0), $[\Sigma\Xi]$(I=3/2)
10	$[N\Sigma]$(I=3/2), $[\Xi\Xi]$(I=0)
$\mathbf{8}_a$	$[N\Xi]$(I=0)

attractive

repulsive

Summary and Outlook

◎Generation of gauge configuration on physical point at Fugaku

$$1/a \simeq 2339[\mathrm{MeV}]$$

$$m_\pi \simeq 137[\mathrm{MeV}]$$

$$m_K \simeq 502[\mathrm{MeV}]$$

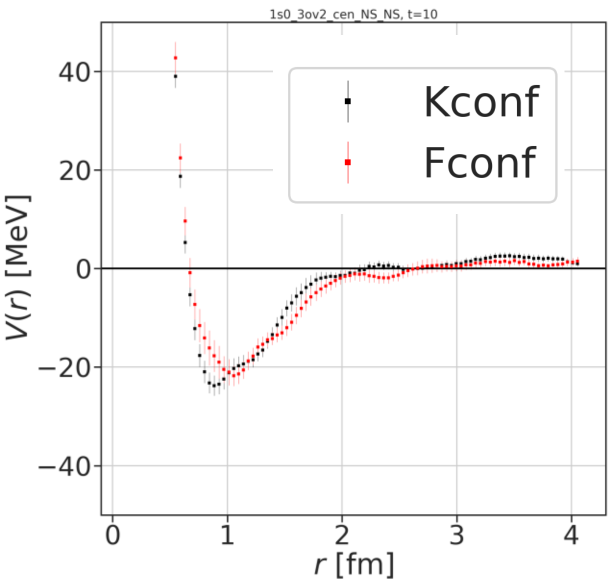
$$m_N \simeq 940[\mathrm{MeV}]$$

Preliminary (1600 configurations)

Future

- topological charge
- Continuum limit

◎Hadron interaction



- hadron interactions are calculated on physical point
- We see (light) quark-mass dependence.
- We must subtract the contamination from inelastic excited states for noisy cannel, e.g., NΛ-NΣ.

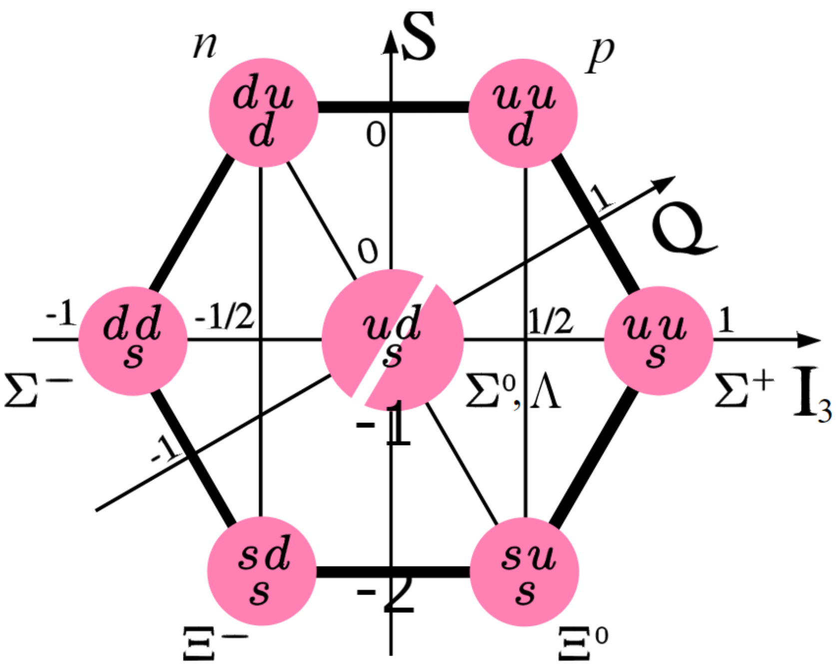
We must establish the way of subtraction, then it will be applied to NΛ-NΣ potential.

Appendix

Target in this study:

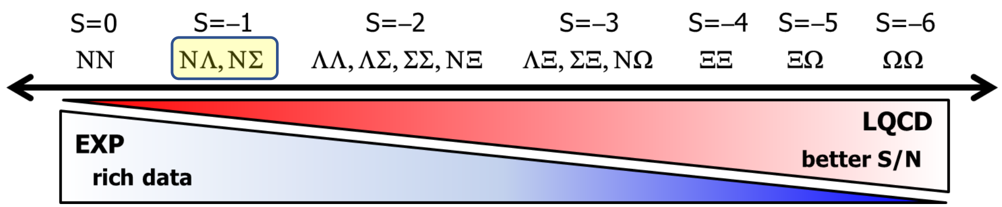
Baryon-Baryon interactions in S=-1 channel

☞ S=-1: NΛ and NΣ (coupled) channel potentials



Baryon mass (from PDG)

		M_B [MeV]
isospin ave. Nucleon	N	938.92 (938.27+939.57)/2
Lambda	Λ	1115.68
isospin ave. Sigma	Σ	1193.15 (1192.64+1189.37+1197.45)/3
isospin ave. Xi	Ξ	1318.29 (1314.86+1321.71)/2



On physical-point lattice QCD configuration !

We calculated the Baryon-baryon interaction using the gauge configuration.
(Results of other channels will be reported by other collaborators in future.)

$\Xi\Lambda - \Xi\Sigma$

coupled channel potential

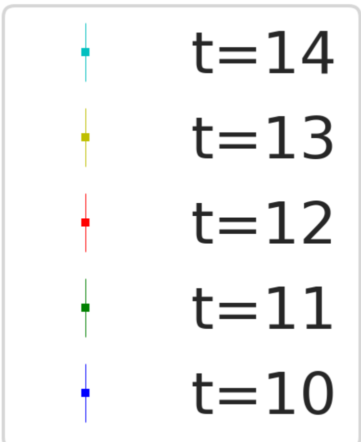
3S1-3D1, I=1/2

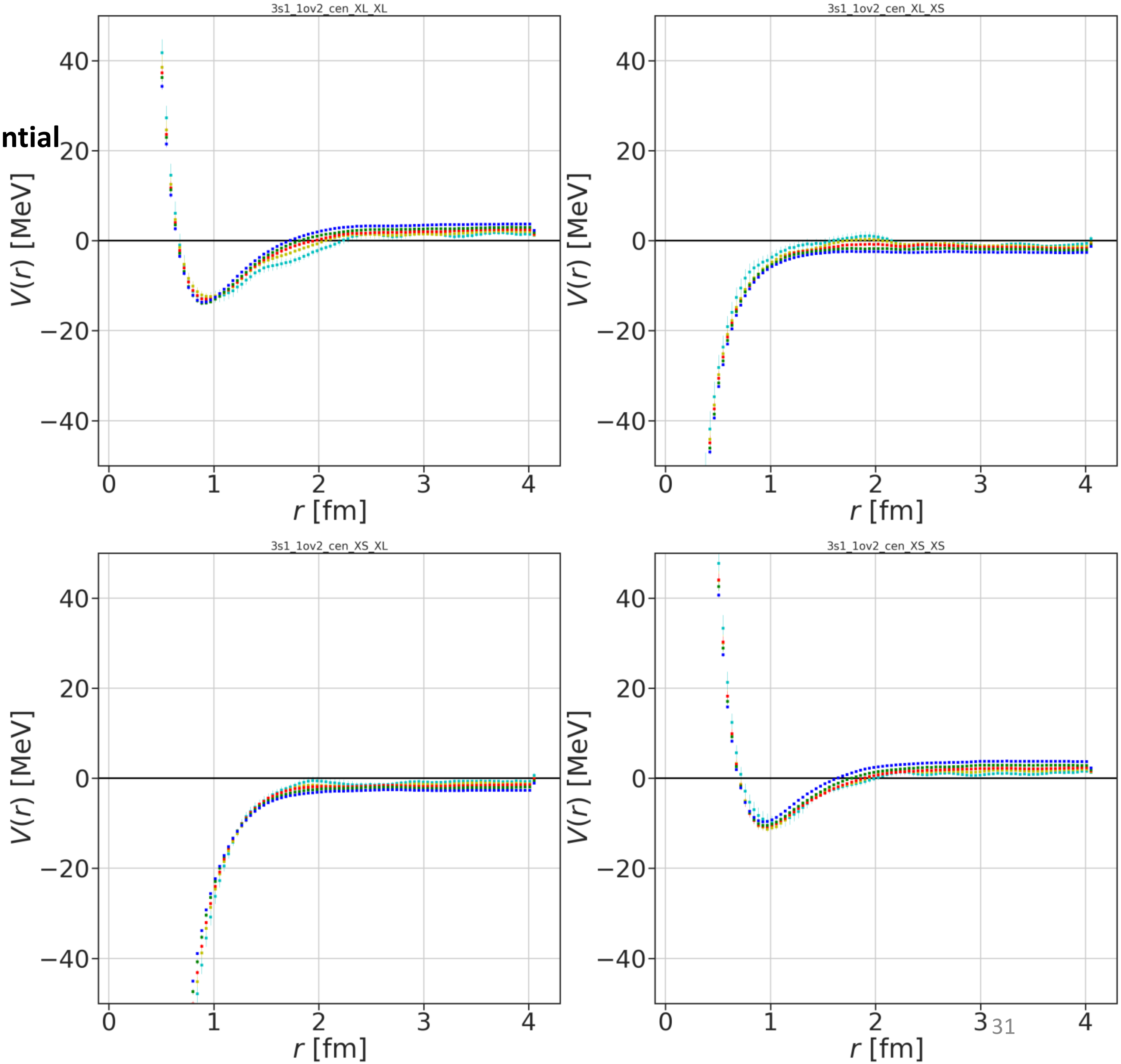
central

binsize=80

Nconf=800

w/ Misner





$\Xi\Lambda - \Xi\Sigma$

coupled channel potential

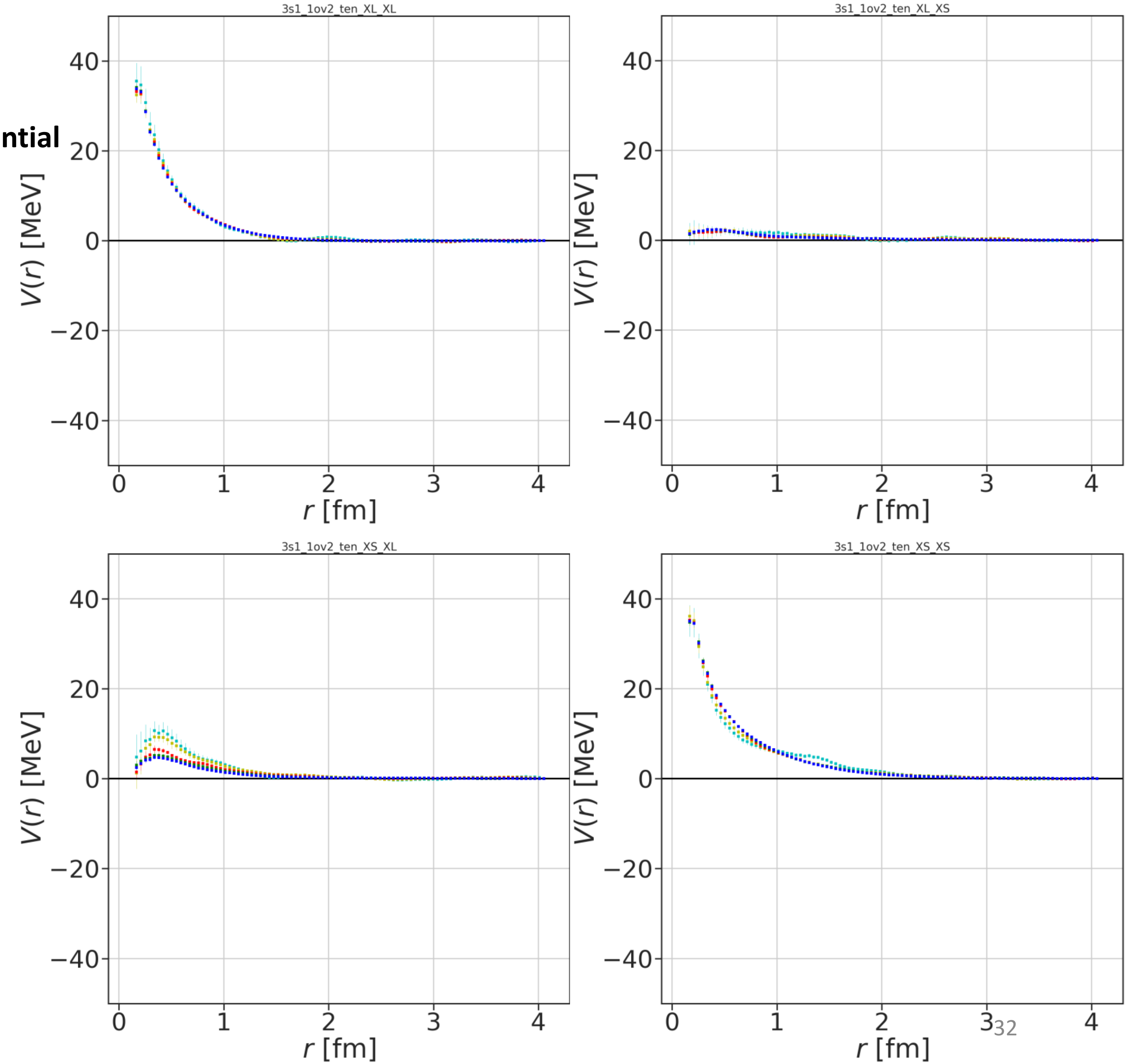
3S1-3D1, I=1/2

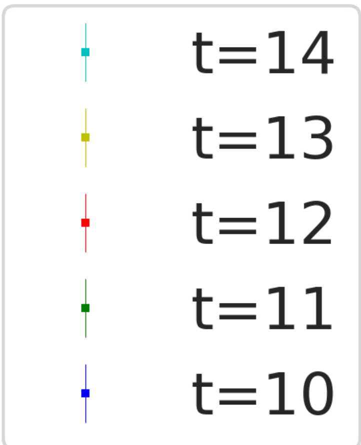
tensor

binsize=80

Nconf=800

w/ Misner





$\Xi\Sigma$ **potential**

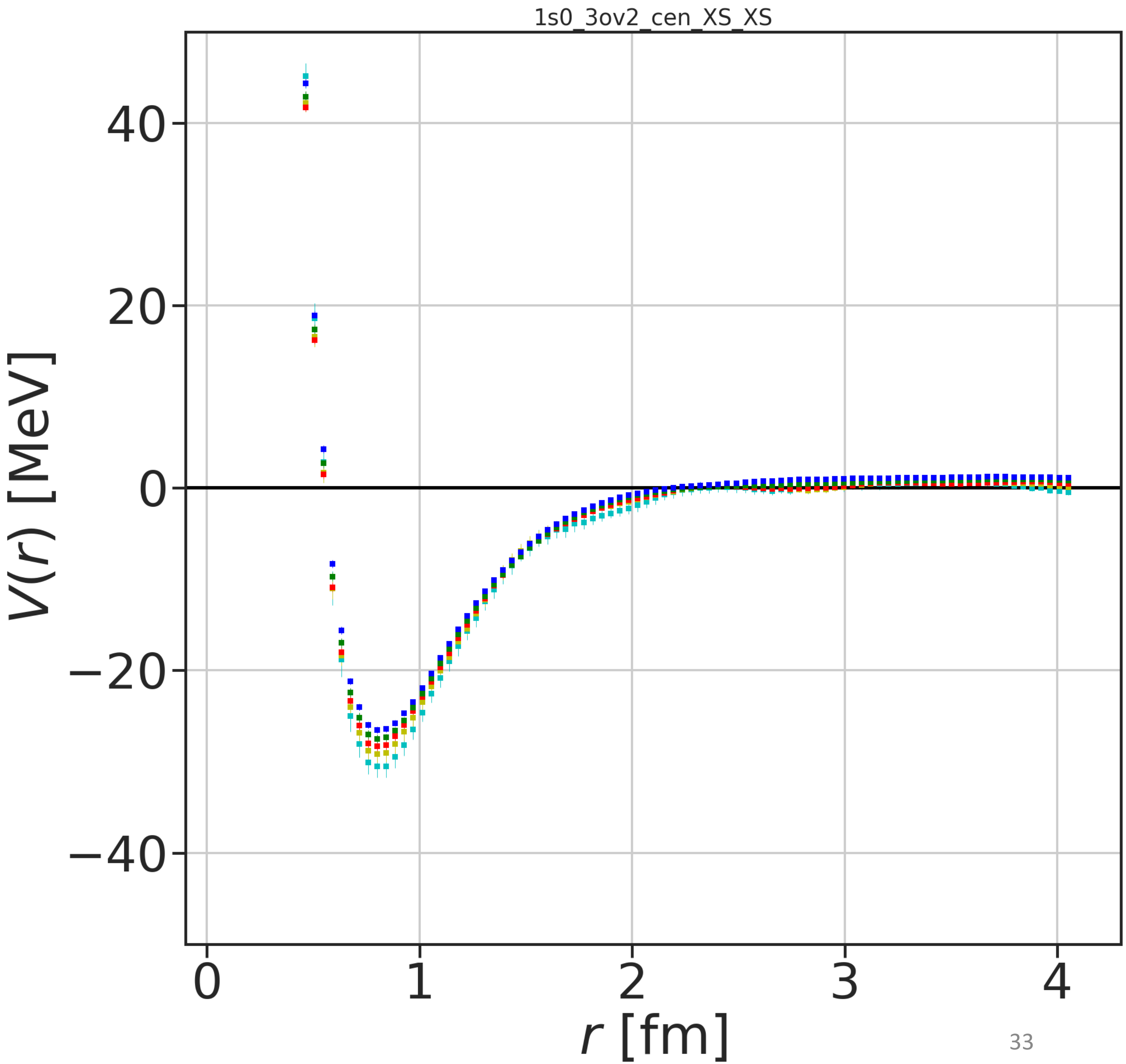
1S0, I=3/2

central

binsize=80

Nconf=800

w/ Misner



ΞΣ potential

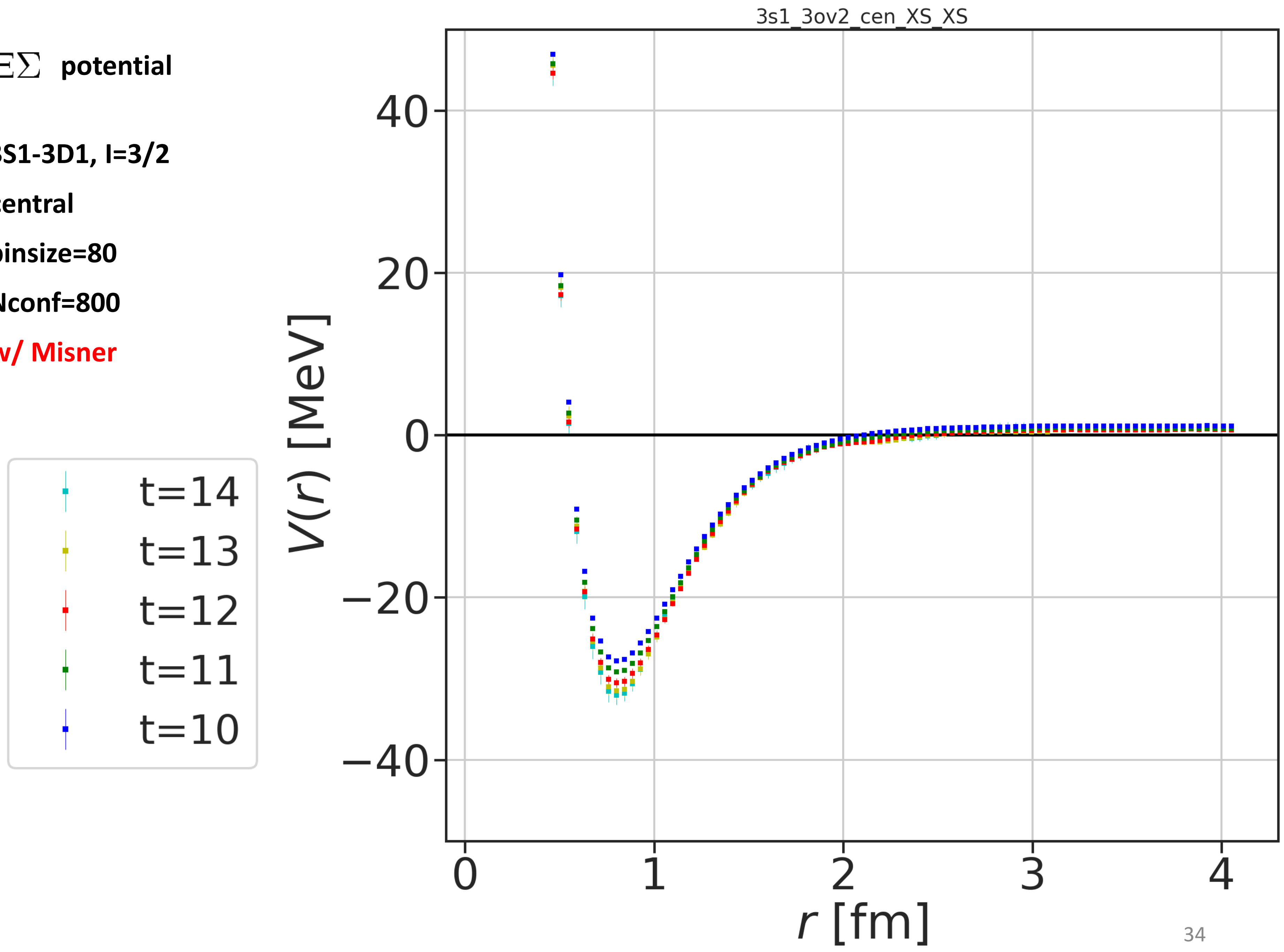
3S1-3D1, I=3/2

central

binsize=80

Nconf=800

w/ Misner



ΞΣ potential

3S1-3D1, I=3/2

tensor

binsize=80

Nconf=800

w/ Misner