# Structure and production of hypernuclei with antisymmetrized molecular dynamics

Masahiro Isaka, Hosei University

# Grand challenges of hypernuclear physics

## Interaction: "baryon-baryon interaction"

- **2 body & 3 body interactions among baryons**
  - hyperon (Y) - nucleon (N), Hyperon (Y) - hyperon (Y)
  - YNN, YYN, … etc.

## Structure: "many-body system of nucleons and hyperon"

- **Addition of hyperon as an impurity in (hyper)nuclei**
  - No nuclear Pauli principle
  - YN interaction is different from NN

**Today: "Structure of hypernuclei"**

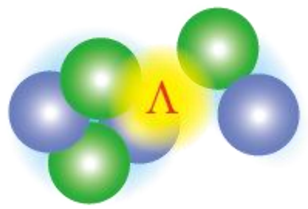
# Structure of Λ hypernuclei

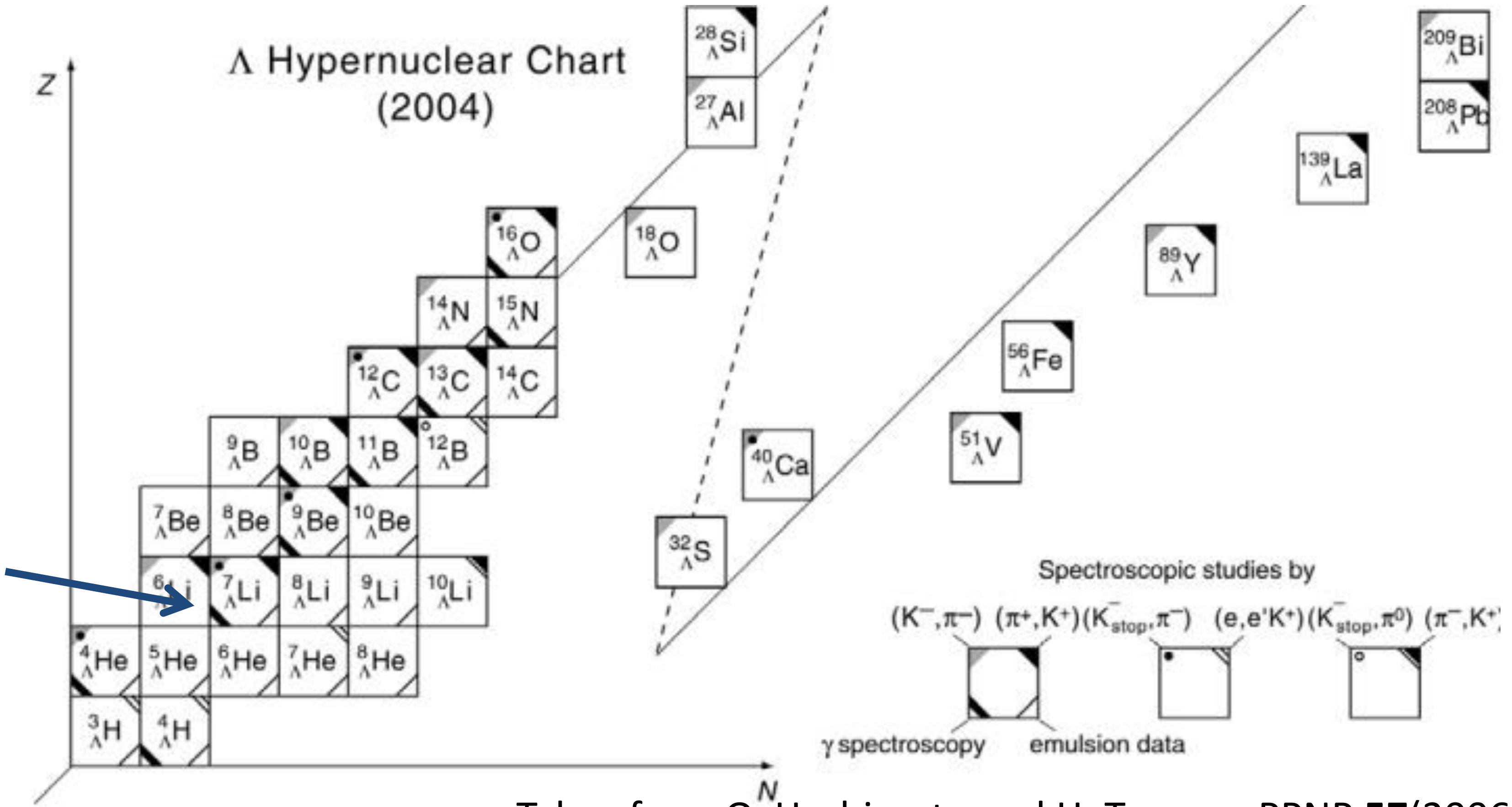
## ◆ Λ hypernuclei observed so far

- Concentrated in light Λ hypernuclei
- Most have well-developed cluster structure

**Light Λ hypernuclei**

**Developed cluster**



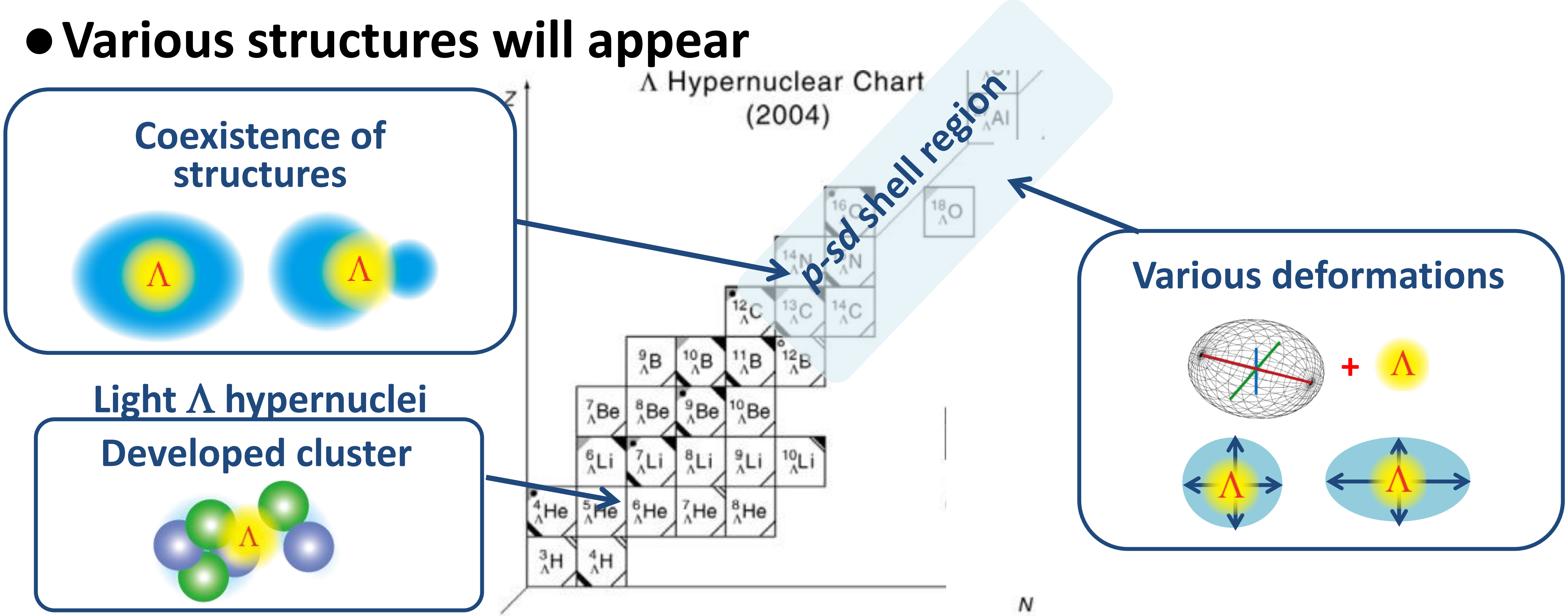




Taken from O. Hashimoto and H. Tamura, PPNP **57**(2006),564.

# Toward heavier and exotic Λ hypernuclei

## ◆Experiments at J-PARC, JLab, *etc.*

- **Heavier(*sd*-shell) hypernuclei can be produced**
- **Various structures will appear**



# What is expected when a $\Lambda$ particle is added to nuclei ?

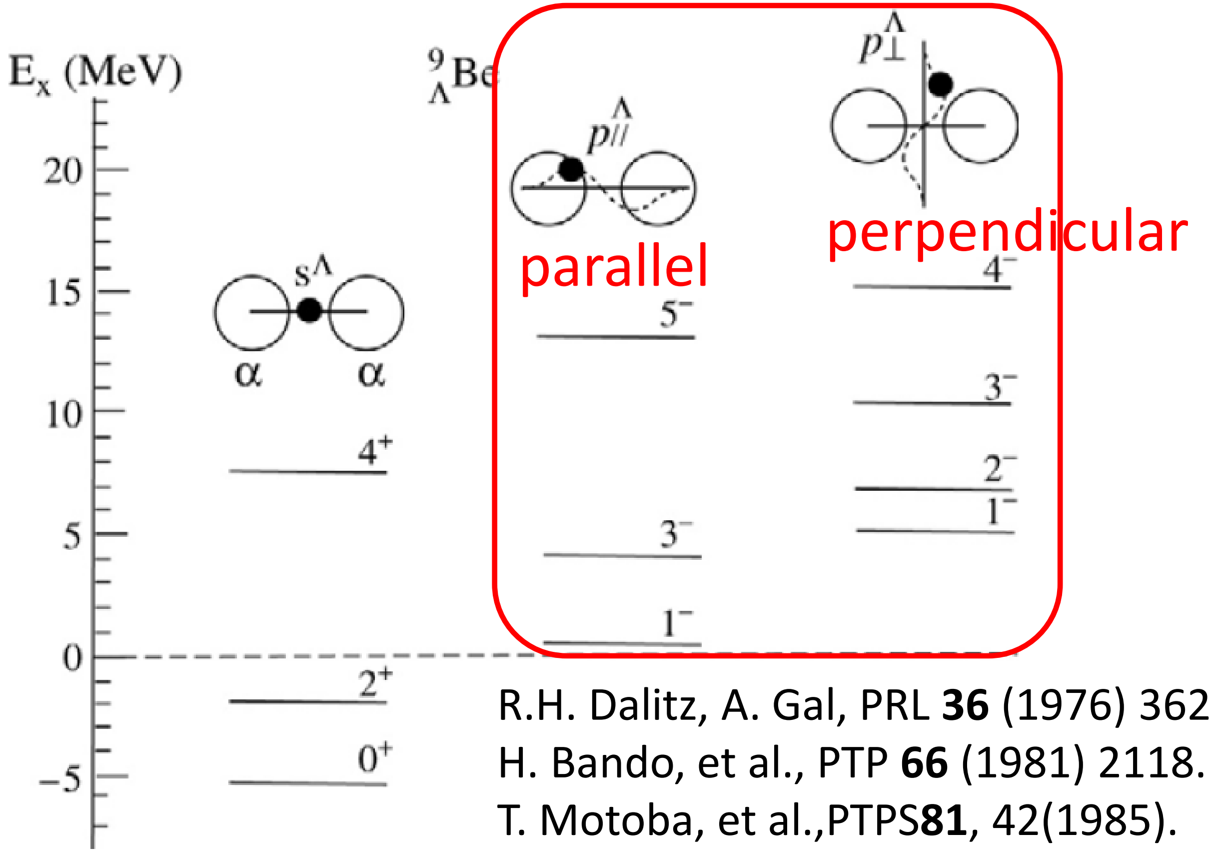
- **Shrinkage and/or deformation change**
  - $\Lambda$ particle can change nuclear radius/deformation
- **Difference of $B_\Lambda$ depending on nuclear structure**
  - Energy shifts in excitation spectra
- **Coupling of $\Lambda$ to deformed nuclei shows unique structure**
  - For example, rotational band, mixing of configuration, … etc.

# Genuine hypernuclear states in $^{9}_{\Lambda}$Be

$^{9}_{\Lambda}$Be: axially symmetric 2α clustering

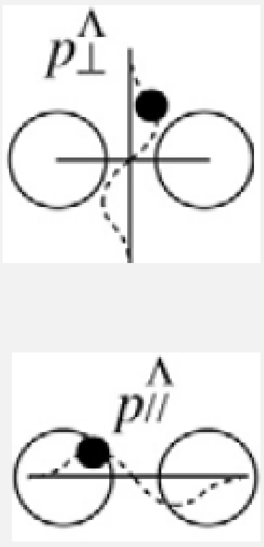
Two rotational bands as *p*-states
- Anisotropic *p* orbit of Λ hyperon
- Axial symmetry of 2α clustering

→ *p*-orbit parallel to/perpendicular to the 2α clustering




R.H. Dalitz, A. Gal, PRL **36** (1976) 362.
H. Bando, et al., PTP **66** (1981) 2118.
T. Motoba, et al.,PTPS**81**, 42(1985).

**p states in $^{9}_{\Lambda}$Be**



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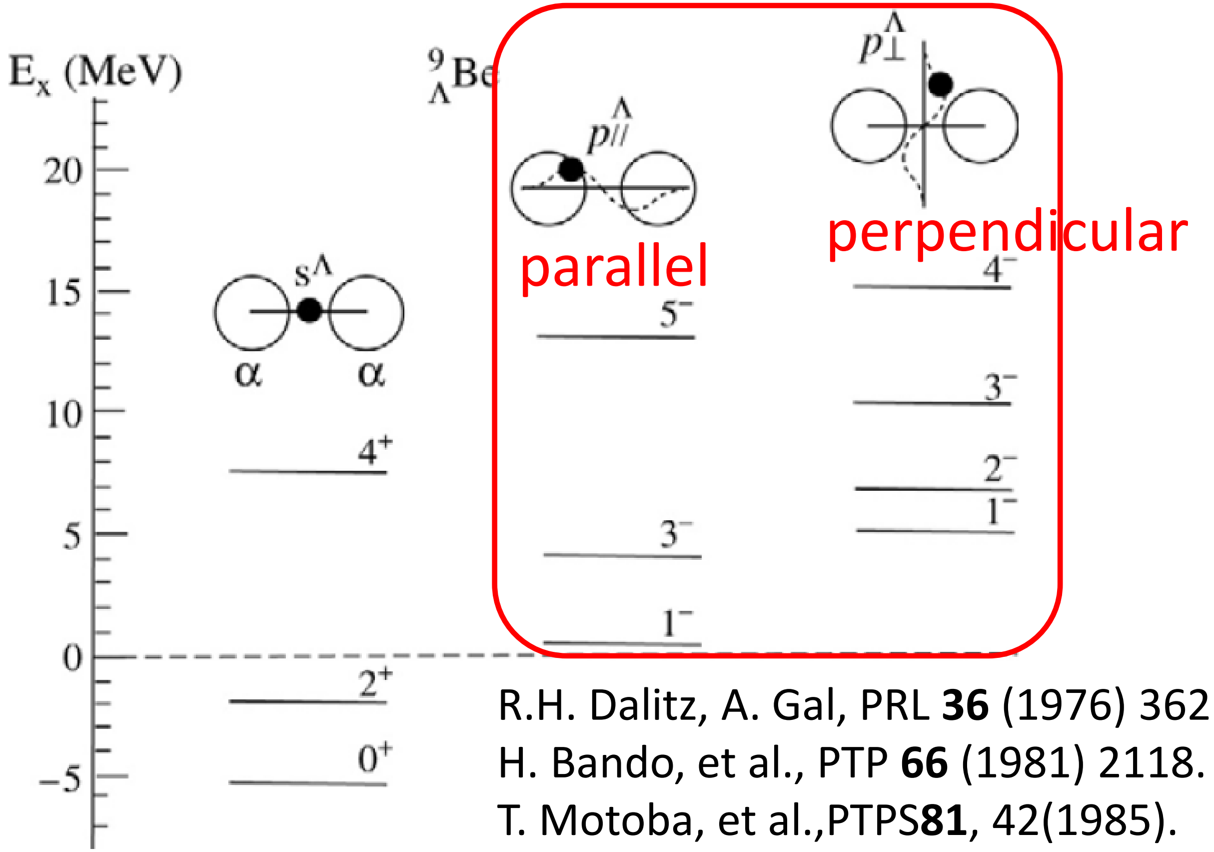


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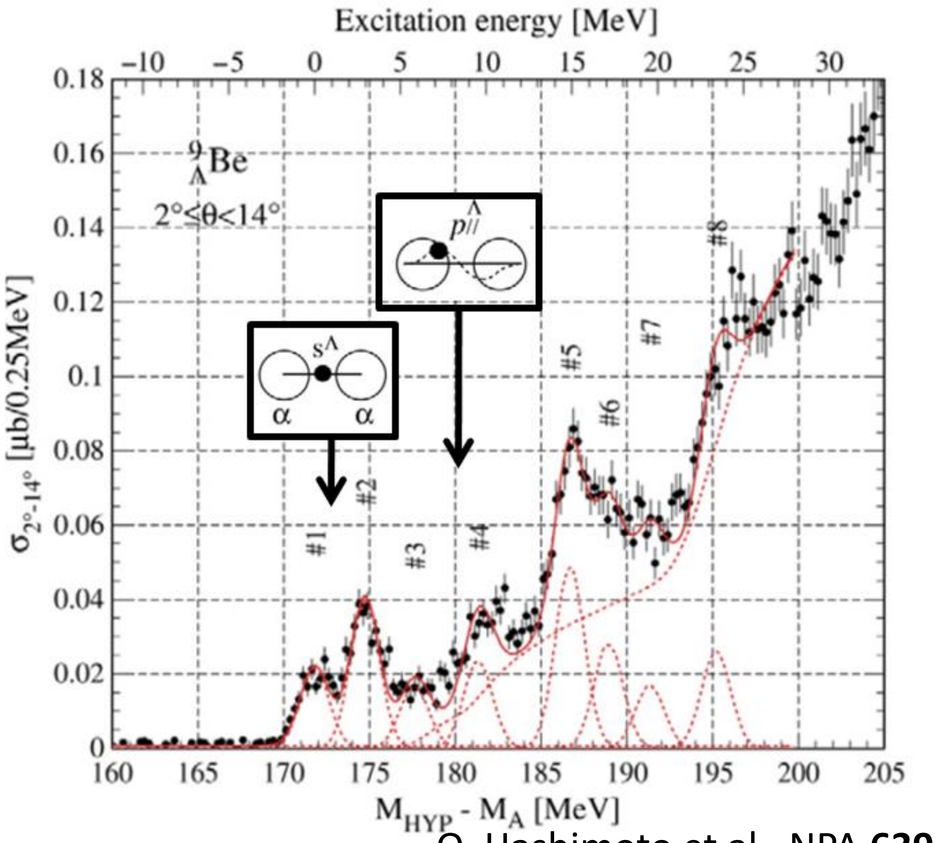


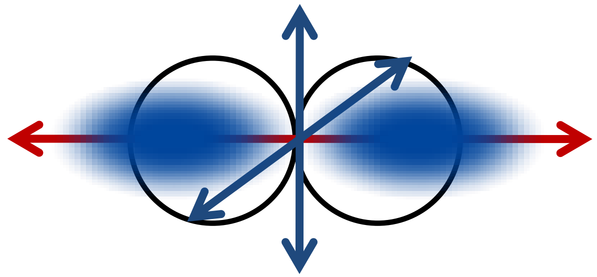

O. Hashimoto et al., NPA **639** (1998) 93c.

# Split of $p$-state in $^{9}_{\Lambda}$Be

$^{9}_{\Lambda}$Be: axially symmetric $2\alpha$ clustering

**Genuine hypernuclear states:**

***p* orbit parallel to $2\alpha$ (long axis)**

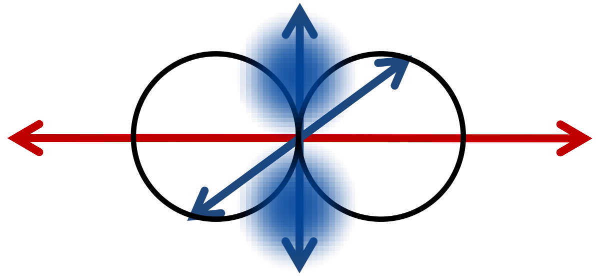


**Large overlap**

**Deeply bound**

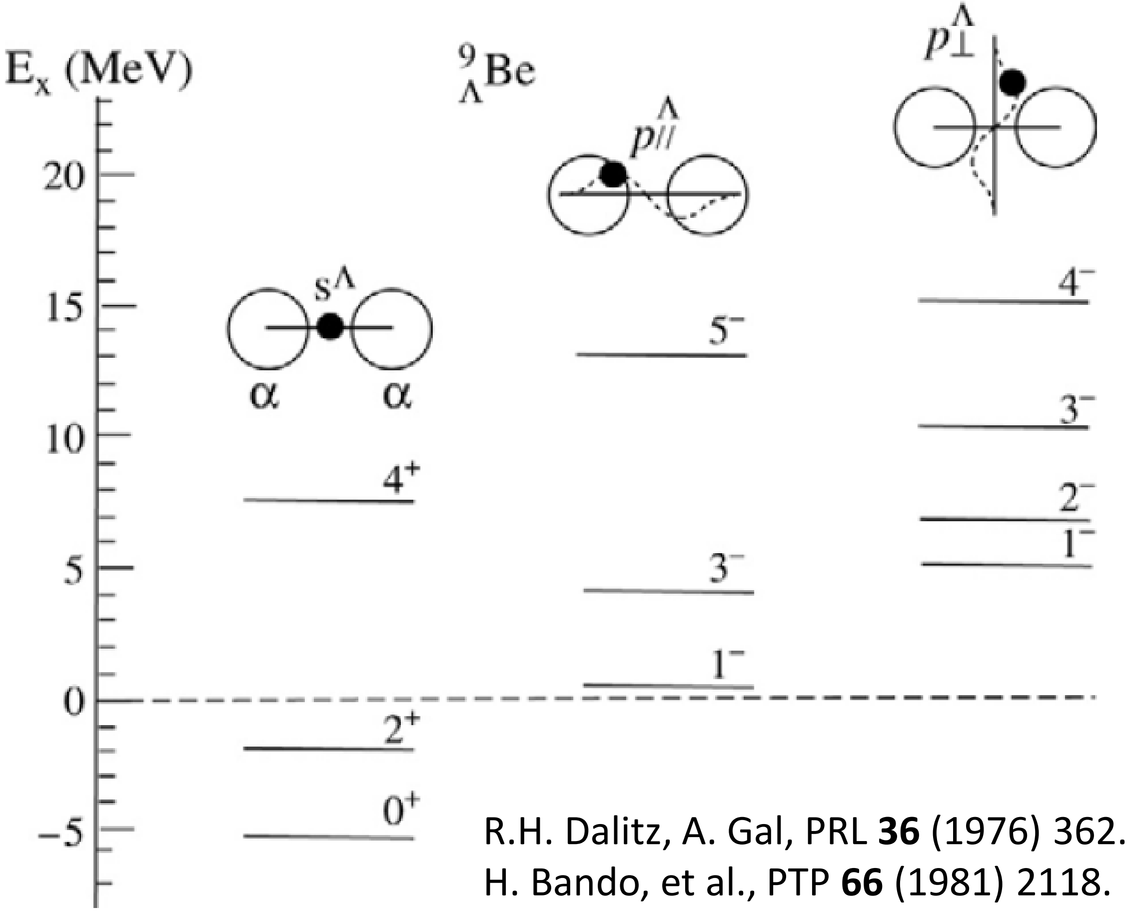
**$^{9}$Be analog states:**

***p* orbit perpendicular to $2\alpha$ (short axes)**



**Small overlap**

**Shallowly bound**

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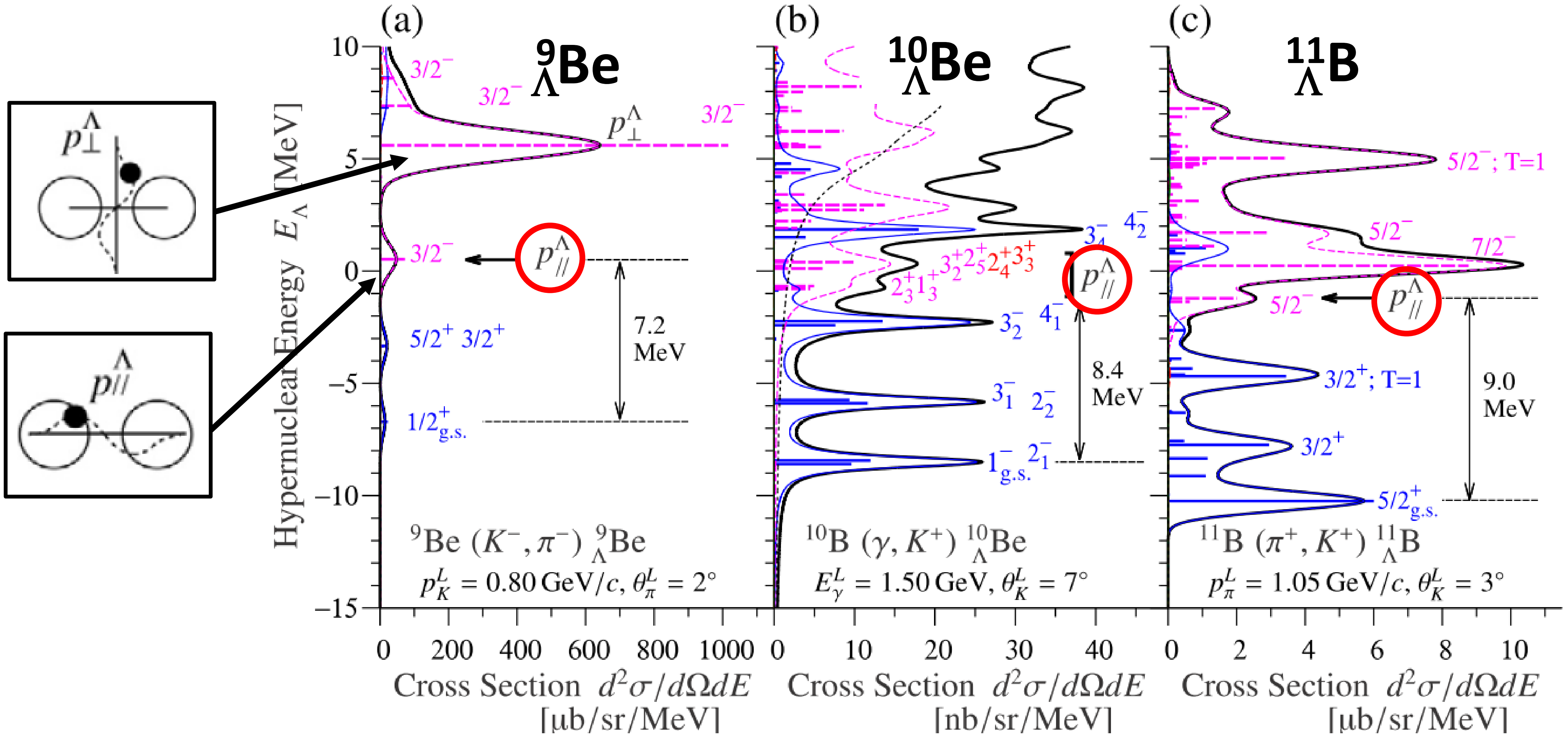
***p*-states splits into 2 bands depending on the direction of *p*-orbits**

# Genuine hypernuclear states in the other hypernuclei

**Genuine hypernuclear states are predicted not only in $^{9}_{\Lambda}$Be but $^{10}_{\Lambda}$Be & $^{11}_{\Lambda}$Be**

Shell model + DWIA calc. by Umeya et al., EPJ Web of Conferernce **271**, 01010(2022)



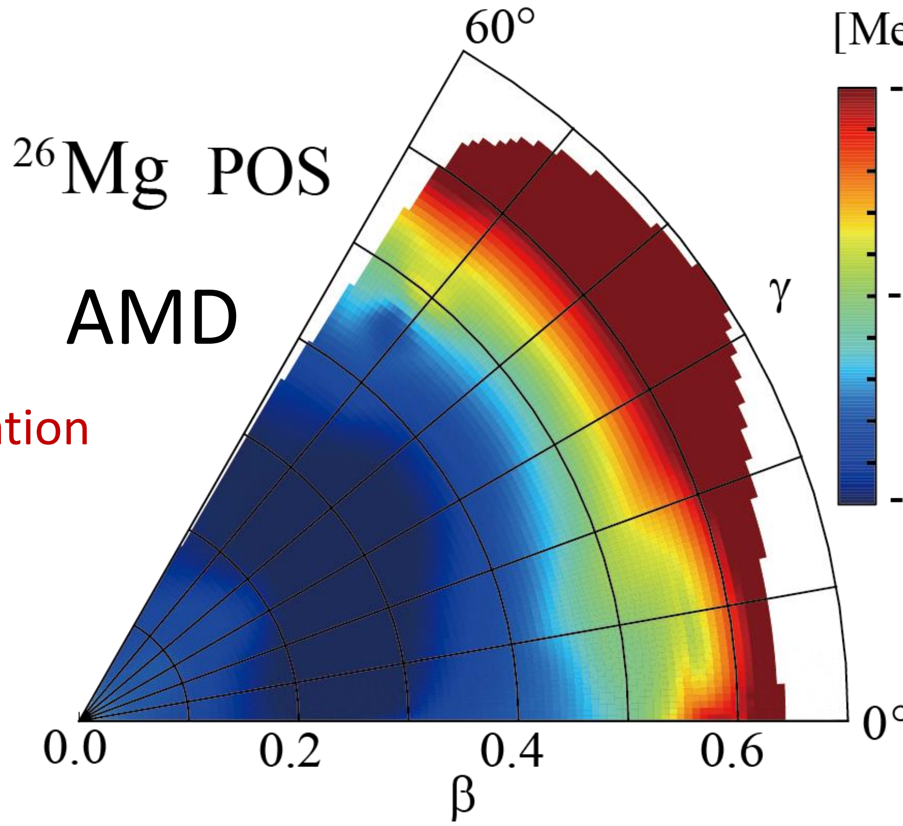
# Coupling of Λ in *p* orbit to triaxially deformed nuclei

$^{26}Mg$

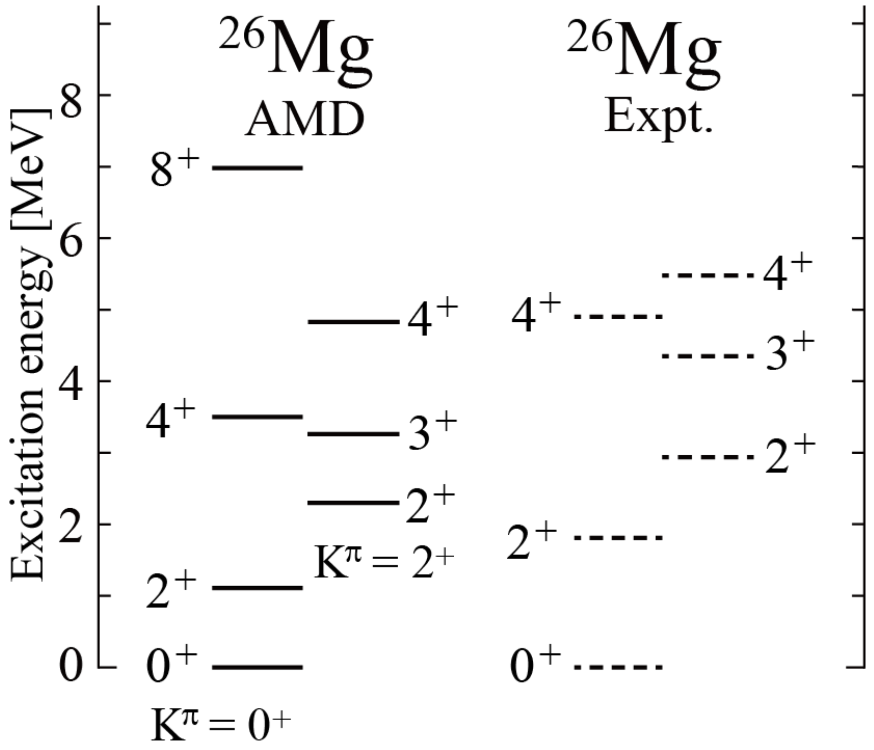
- Shell gap in Nilsson diagram: Z=12 (prolate) vs. N=14 (oblate) → triaxial
- β,γ-softness is discussed by several authors

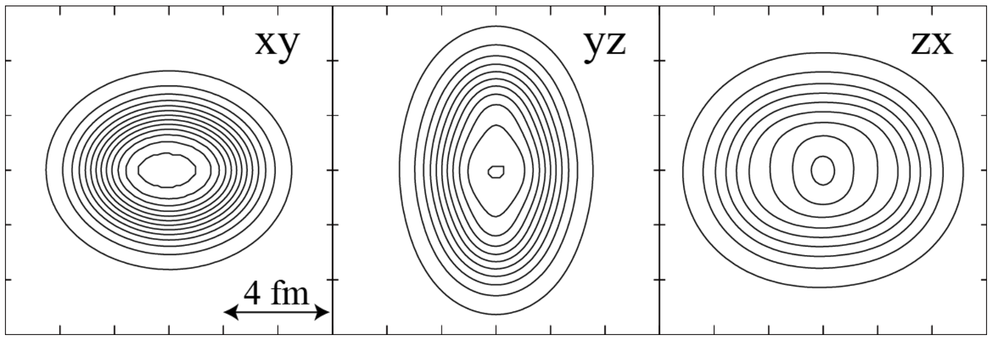
Terasaki et al. NPA**621**(1997)
Rodrıguez-Guzman et al. NPA**709** (2002)
Peru et al PRC**77** (2008)
Hinohara, Kanada-En'yo PRC**83** (2011)

AMD




β: degree of deformation
γ: (tri)axiality

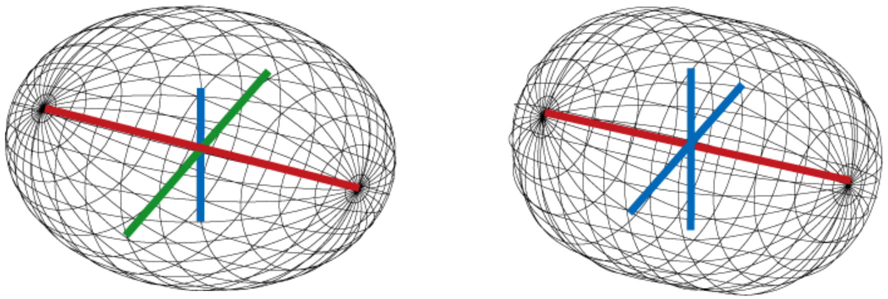


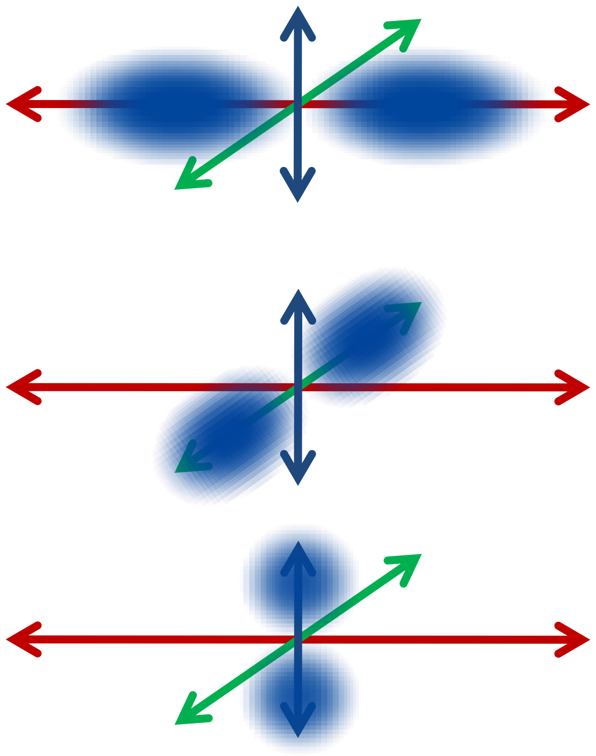

**What will happen if Λ in p orbit is coupled to triaxially deformed $^{26}Mg$ ?**

# Triaxial deformation

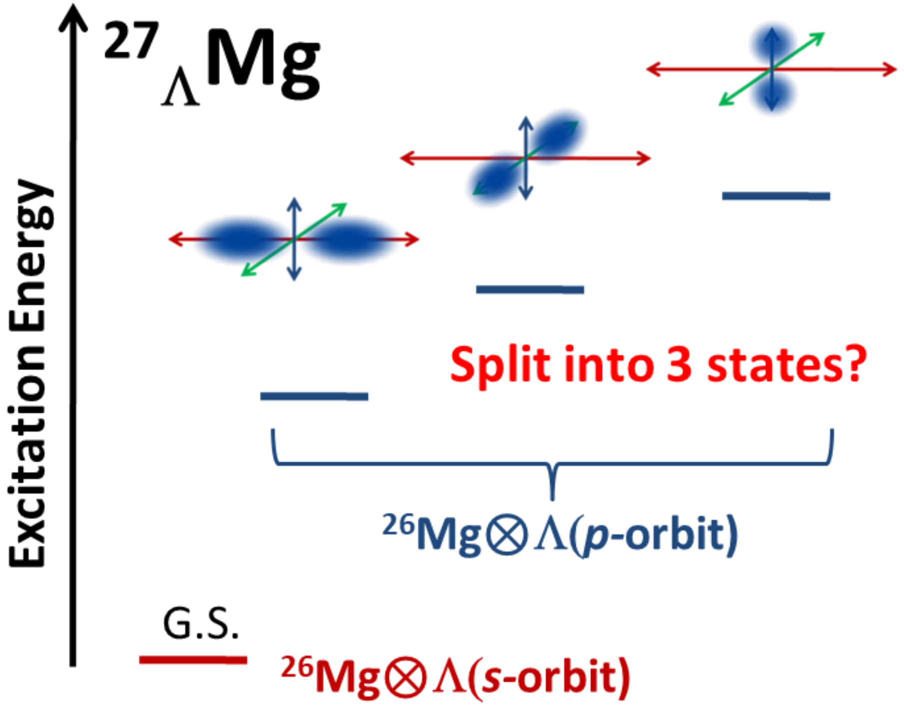
**If nucleus is triaxially deformed,**
***p*-states can split into 3 different state**



Triaxial deformation    Prolate deformation




Candidate: Mg hypernuclei




**Observing the 3 different *p*-states is strong evidence of triaxial deformation**

# Theoretical Framework: HyperAMD

M.I., *et al.*, PRC**83**(2011) 044323
M. I., *et al.*, PRC**83**(2011) 054304

## HyperAMD: antisymmetrized molecular dynamics for hypernuclei

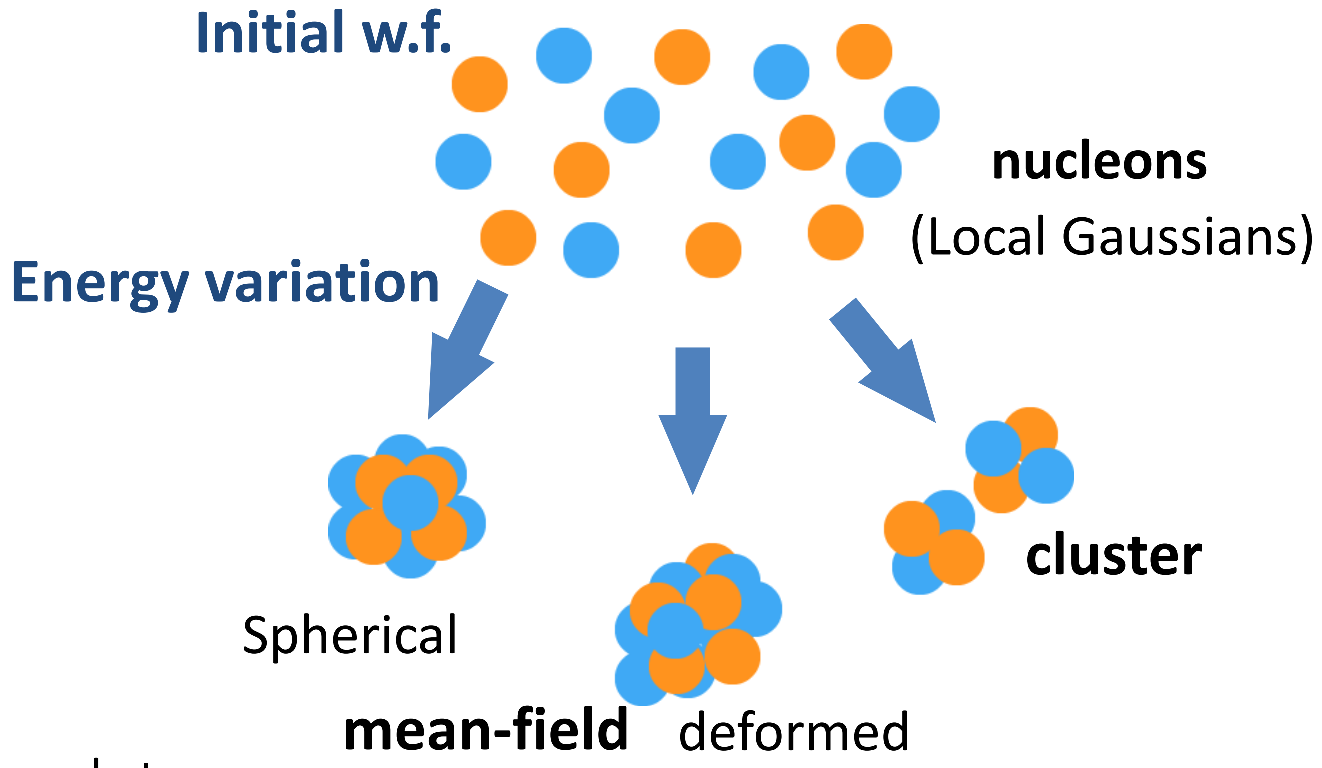
### Hamiltonian

$$\hat{H} = \hat{T}_N + \hat{V}_{NN} + \hat{T}_Y + \hat{V}_{YN} - \hat{T}_g$$

NN : Gogny D1S density functional

YN : YNG ESC14+MBE, $\sigma \cdot \sigma$ & LS are tuned

M. I., Y. Yamamoto, T. Motoba, PRC**101** (2020)




### Model wave function

$$\psi(\vec{r}) = \sum_m c_m \phi_m(r_Y) \otimes \frac{1}{\sqrt{A!}} \det[\phi_i(\vec{r}_j)]$$

Nuclear part : Slater determinant of nucleon wave packets

Hyperon wave function : Superposition of local Gaussians

$\Lambda$ particle is also optimized through the energy variation

# Theoretical Framework: HyperAMD

M.I., *et al.*, PRC**83**(2011) 044323
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## ◆Procedure of the numerical calculation

**Variation**

- Imaginary time development method: $\frac{dX_i}{dt} = \frac{\kappa}{\hbar}\frac{\partial H^{\pm}}{\partial X_i^{*}}$ $\kappa < 0$
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i$

⬇

**Angular Momentum Projection**

$$\left|\Phi^{s}_{K}; JM\right\rangle = \int d\Omega D^{J*}_{MK}(\Omega) R(\Omega) \left|\Phi^{s+}\right\rangle$$

⬇

**Generator Coordinate Method (GCM)**

- Superposition of intrinsic wave functions with different configuration
- Diagonalization of $H^{J\pm}_{sK,s'K'}$ and $N^{J\pm}_{sK,s'K'}$

$$H^{J\pm}_{sK,s'K'} = \left\langle \Phi^{s}_{K}; J^{\pm}M \left| \hat{H} \right| \Phi^{s'}_{K'}; J^{\pm}M \right\rangle$$

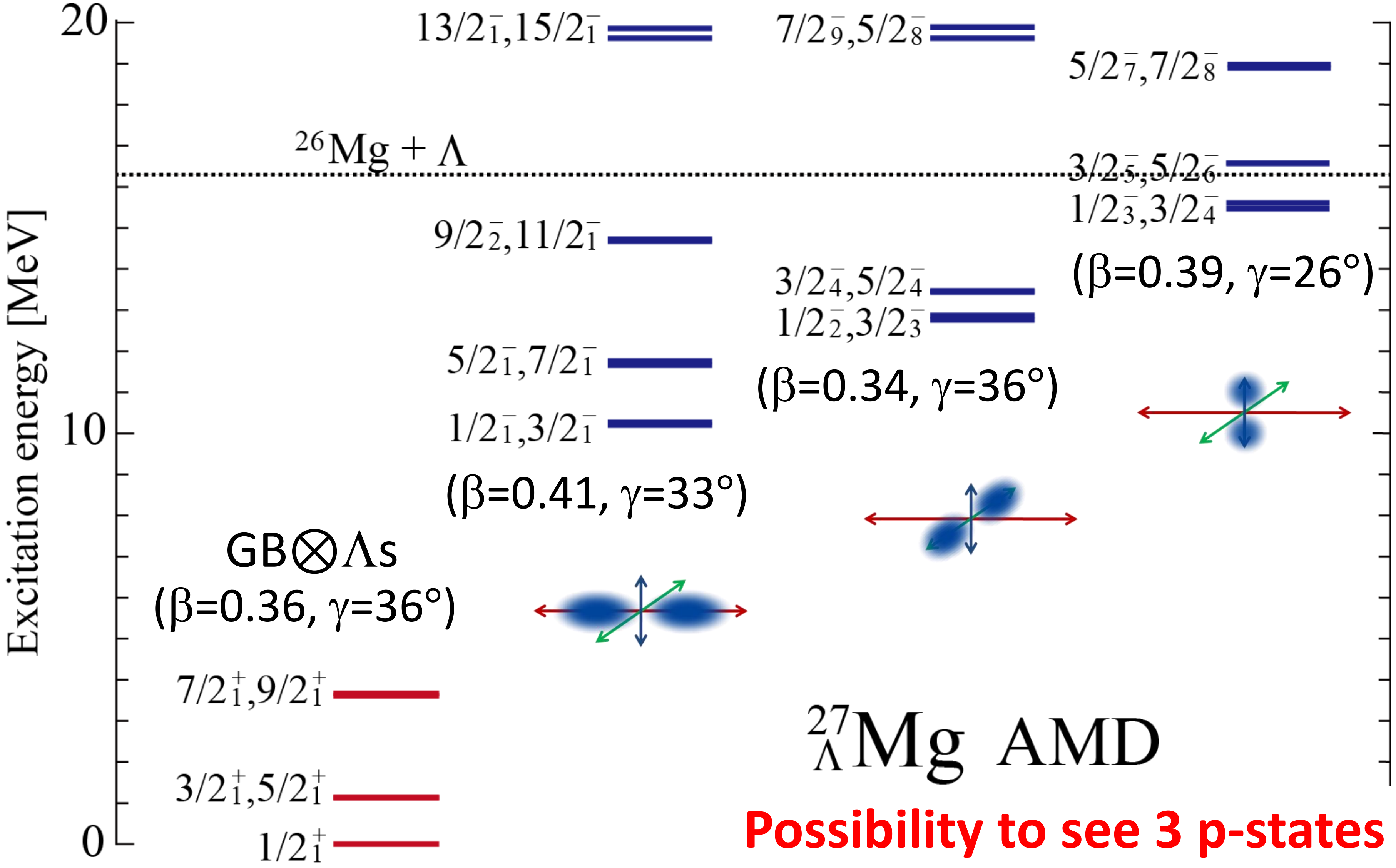
$$N^{J\pm}_{sK,s'K'} = \left\langle \Phi^{s}_{K}; J^{\pm}M \middle| \Phi^{s'}_{K'}; J^{\pm}M \right\rangle$$

$$\left|\Psi^{J\pm M}\right\rangle = \sum_{sK} g_{sK} \left|\Phi^{s}_{K}; J^{\pm}M\right\rangle$$

# Results: $^{27}{}_{\Lambda}Mg$

M.I., et al., PRC**87**, 021304(R) (2013)
and calculation ongoing

- **3 bands are obtained by Λ in *p*-orbit** → **Splitting of the *p* states**




cf. $^{26}$Mg ground state: β=0.41, γ=33°

**Possibility to see 3 p-states by $^{27}Al(e,e'K+)^{27}{}_{\Lambda}Mg$**

# Production cross section of hypernuclei with HyperAMD

Collaborator: T. Motoba

# Goal and Strategy

**Goal** To extract structure info from hypernuclear production cross section

**Strategy** To analyze production cross section of hypernuclei by describing various structures using HyperAMD wf

- Distortion Wave Impulse Approx. (DWIA)
  → Plane Wave Impulse Approx. (PWIA) as the 1st step
- Applications to suitable reaction to extract structure info
  → $^{27}$Al($\gamma$,K$^+$)$^{27}{}_{\Lambda}$Mg corresponding to $^{27}$Al(e,e'K$^+$)$^{27}{}_{\Lambda}$Mg
- With elementary amplitudes based on isobaric model

# Today's talk

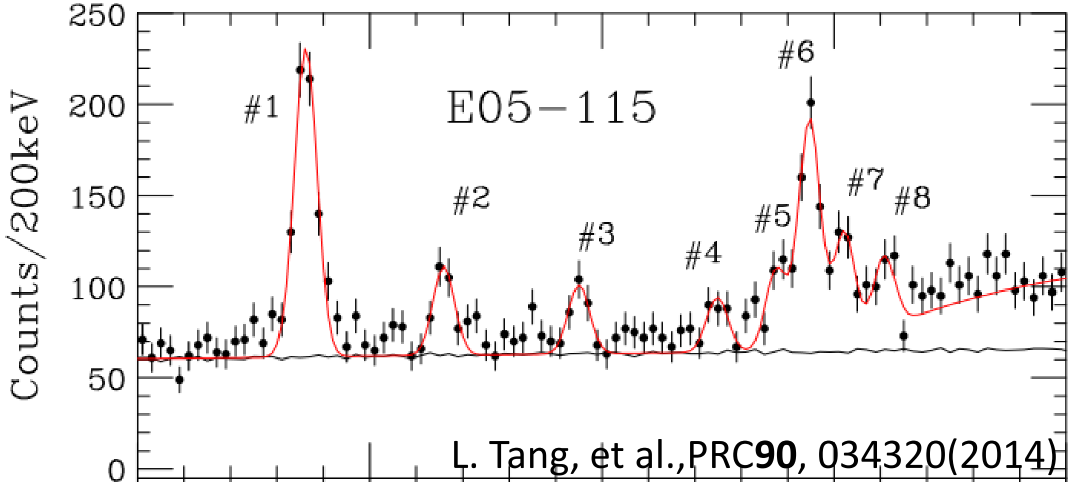
To compare with existing experimental data and other theoretical results

→ $^{12}C(\gamma, K^+)^{12}{}_{\Lambda}B$ as a typical p-shell $\Lambda$ hypernucleus

- Peak structure
  "Large peaks and core excitations"

- Dependence on radius
  "How important is to reproduce nuclear radii?"

$^{12}C(e, e'K^+)^{12}{}_{\Lambda}B$




L. Tang, et al., PRC**90**, 034320(2014)

# Theoretical framework: Structure calculation

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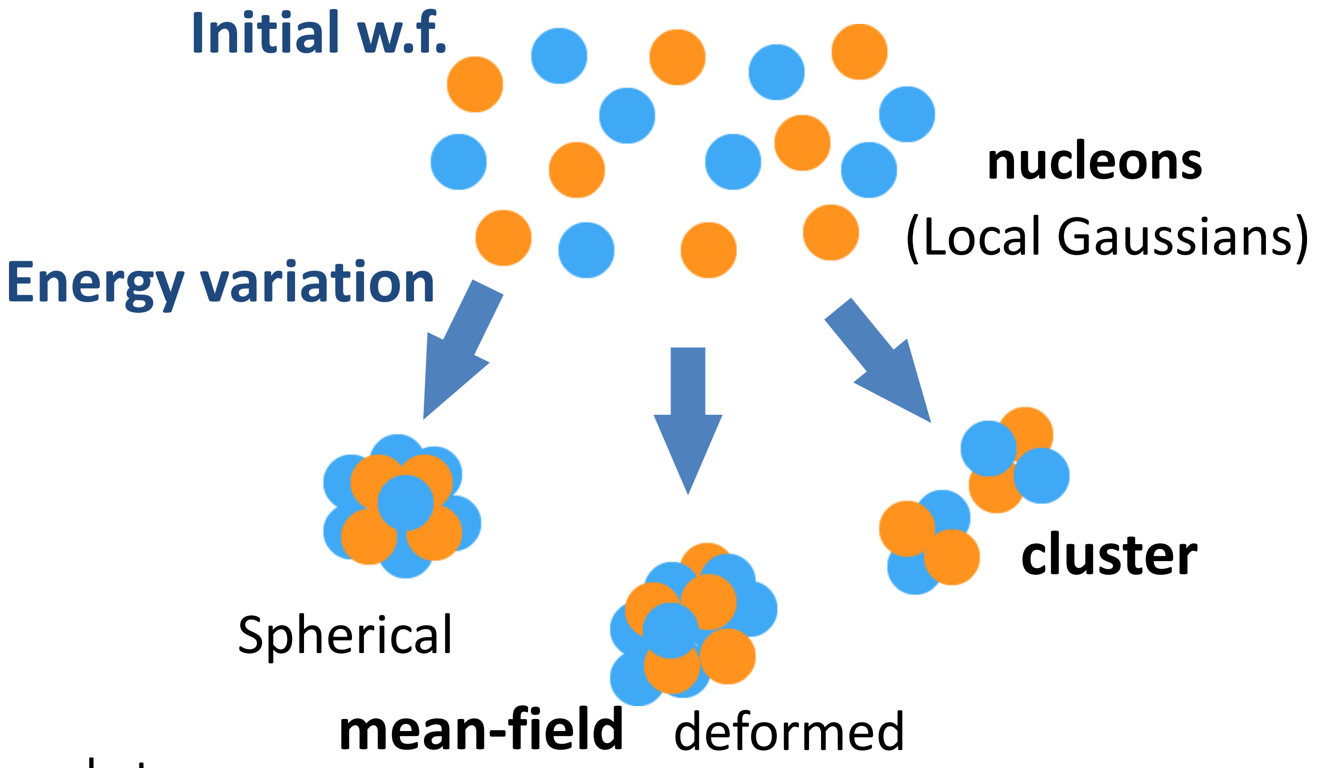
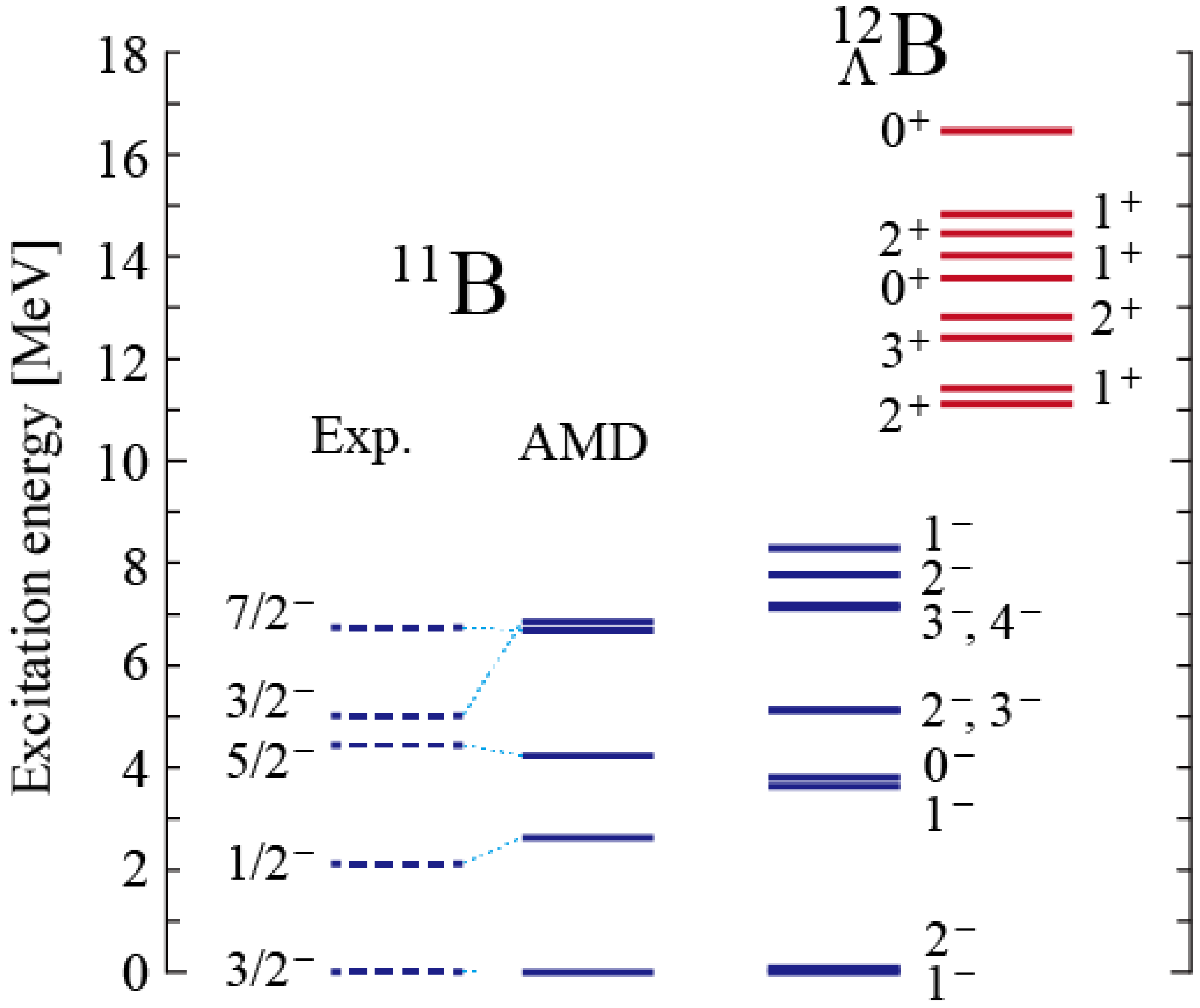



$\Lambda$ particle is also optimized through the energy variation

# Results of structure calculation




$B_\Lambda = 12.1$ MeV

*cf.* $B_\Lambda^{\text{exp}} = 11.38 \pm 0.02$ MeV

$k_F$ is common in all states,
$k_F$ = 1.08 fm$^{-1}$

$^{11}$B : $r_{rms}$ = 2.50 fm
(exp. 2.09 $\pm$ 0.12 fm)

# Theoretical framework: Reaction calculation

- **(γ, K⁺) reaction**

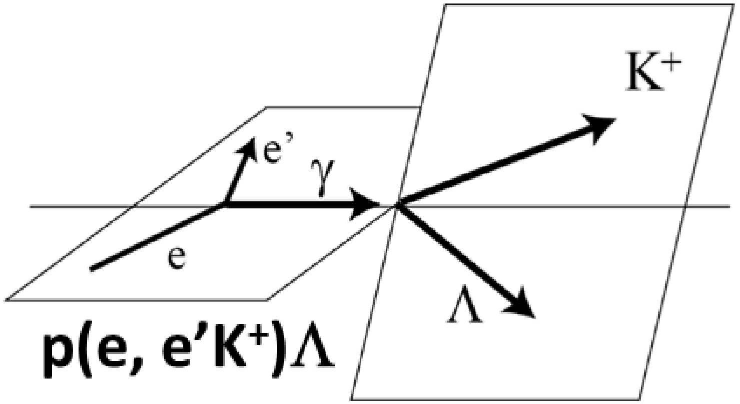
T. Motoba *et al.*, PTP**185**, 224(2010)

$$\frac{d\sigma}{d\Omega}(\theta_K^{\mathrm{Lab}}) = \frac{s p_K^2 E_K E_H}{p_K (E_H + E_K) - E_\gamma E_K \cos\theta_K^{\mathrm{Lab}}} \sum_{M_f} R(fi; M_f),$$

$$R(fi; M_f) = \frac{1}{2J_i + 1} \sum_{M_i} \Psi_{\mathrm{GCM}}^{J_f \pi M_f} |\langle \Psi_{\mathrm{GCM}}^{J_f \pi M_f} | O | \Psi_{\mathrm{GCM}}^{J_i \pi M_i} \rangle|^2$$

AMD + GCM wave functions

**Various structure**




$$O = \int d^3 r \chi_K^{(-)*}(\mathbf{p}, \xi\mathbf{r}) \chi_K^{(+)}(\mathbf{k}, \mathbf{r}) \sum_{j=1}^{A} V_-^{(j)} \delta(\mathbf{r} - \eta \mathbf{r}_j) \langle \mathbf{k} - \mathbf{p}, \mathbf{p} | t | \mathbf{k}, 0 \rangle$$

Plane Wave
as 1st step

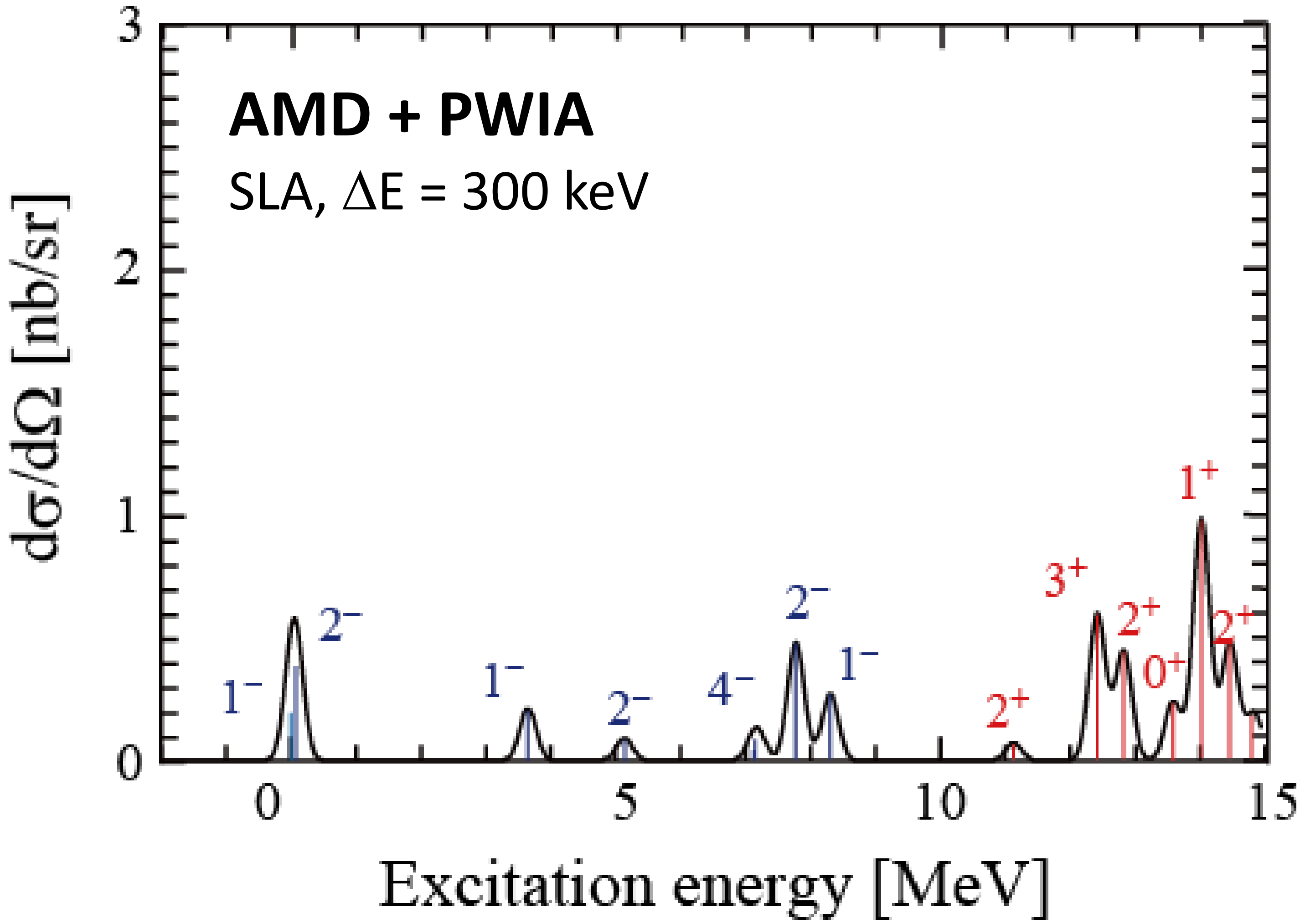
**Elementary amplitude**

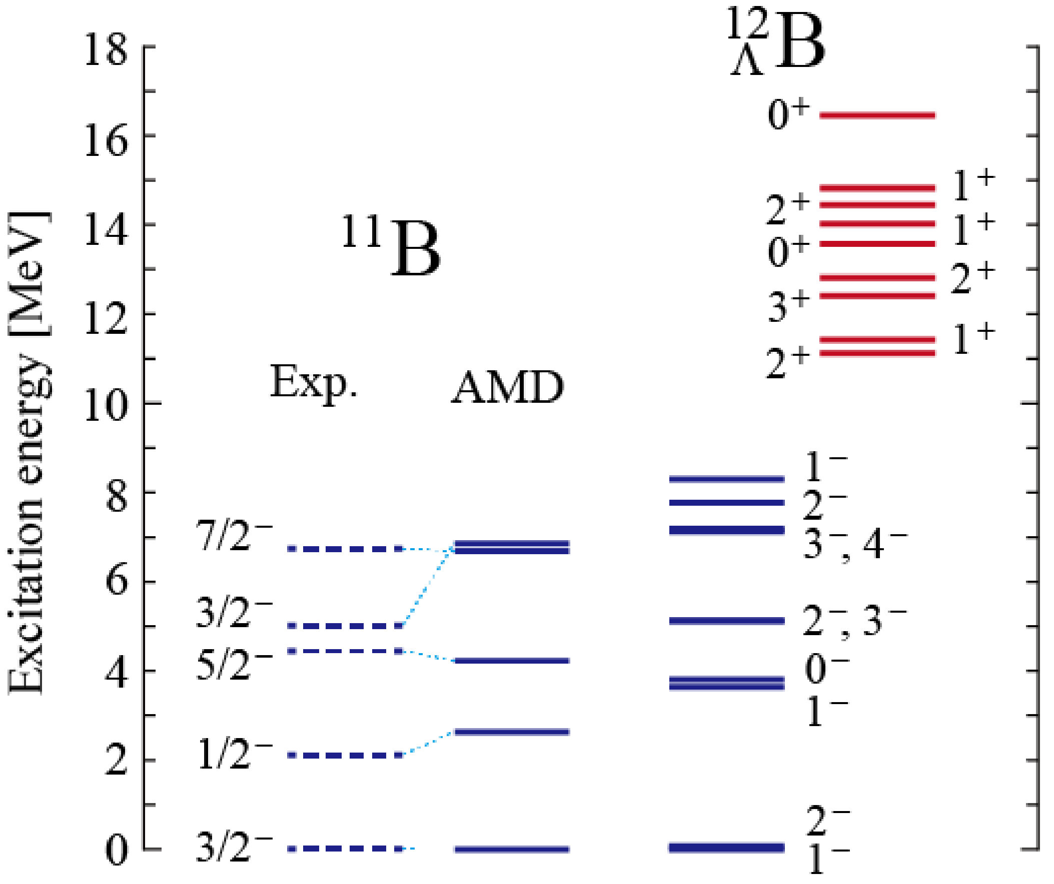
Elementary amplitude: $\langle \boldsymbol{k} - \boldsymbol{p} | t | \boldsymbol{k}, 0 \rangle = \varepsilon_0 (f_0 + g_0 \sigma_0) + \varepsilon_x (g_1 \sigma_1 + g_{-1} \sigma_{-1})$

In this calc., the same version of Saclay-Lyon (SLA) as Motoba et al.,PTP**185**,224(2010)

# Results: Production cross section of $^{12}_{\Lambda}B$

**$^{12}C(\gamma, K^+)^{12}_{\Lambda}B$** $E\gamma$ = 1.3 GeV, $\theta_K$ = 3 deg



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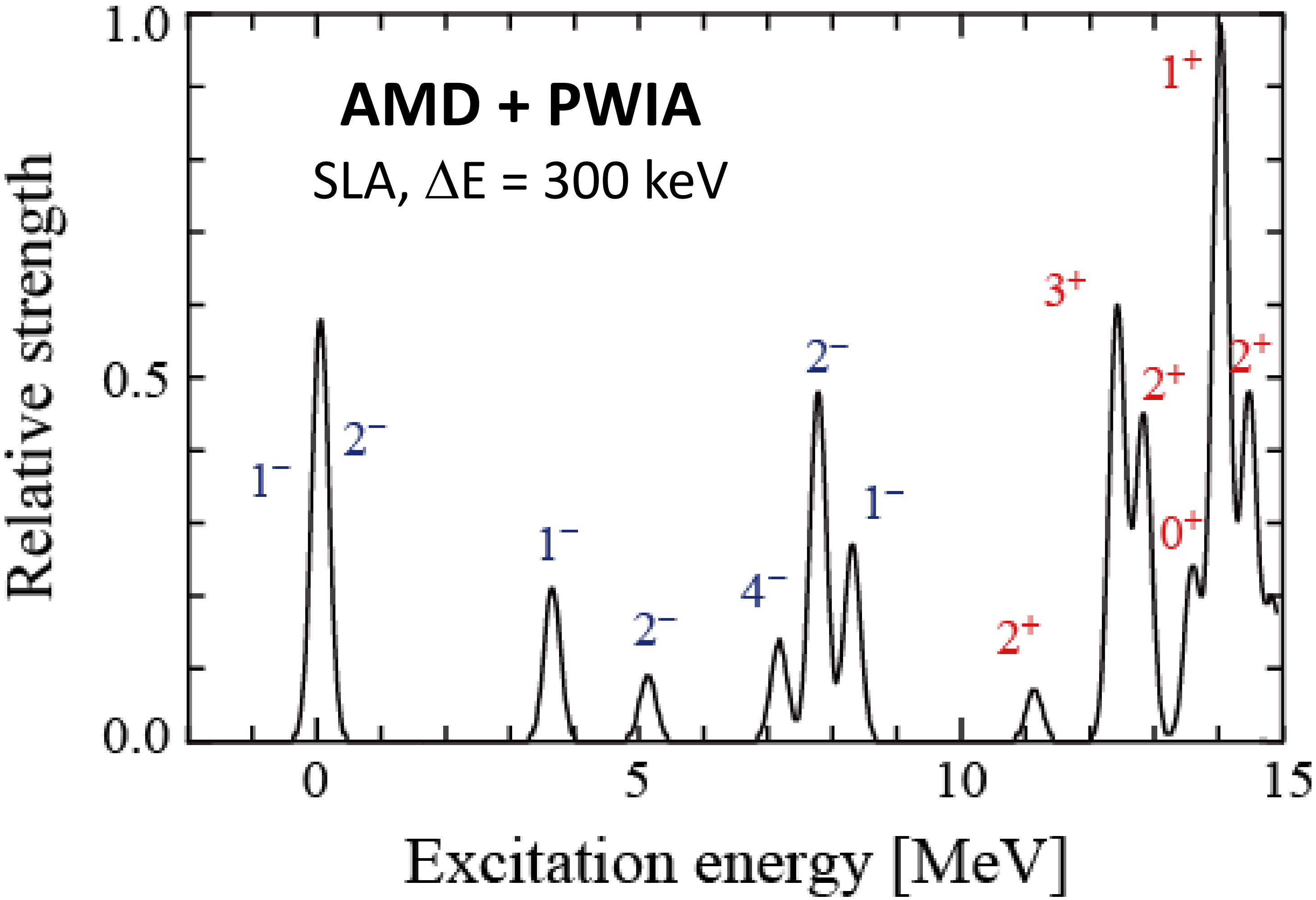


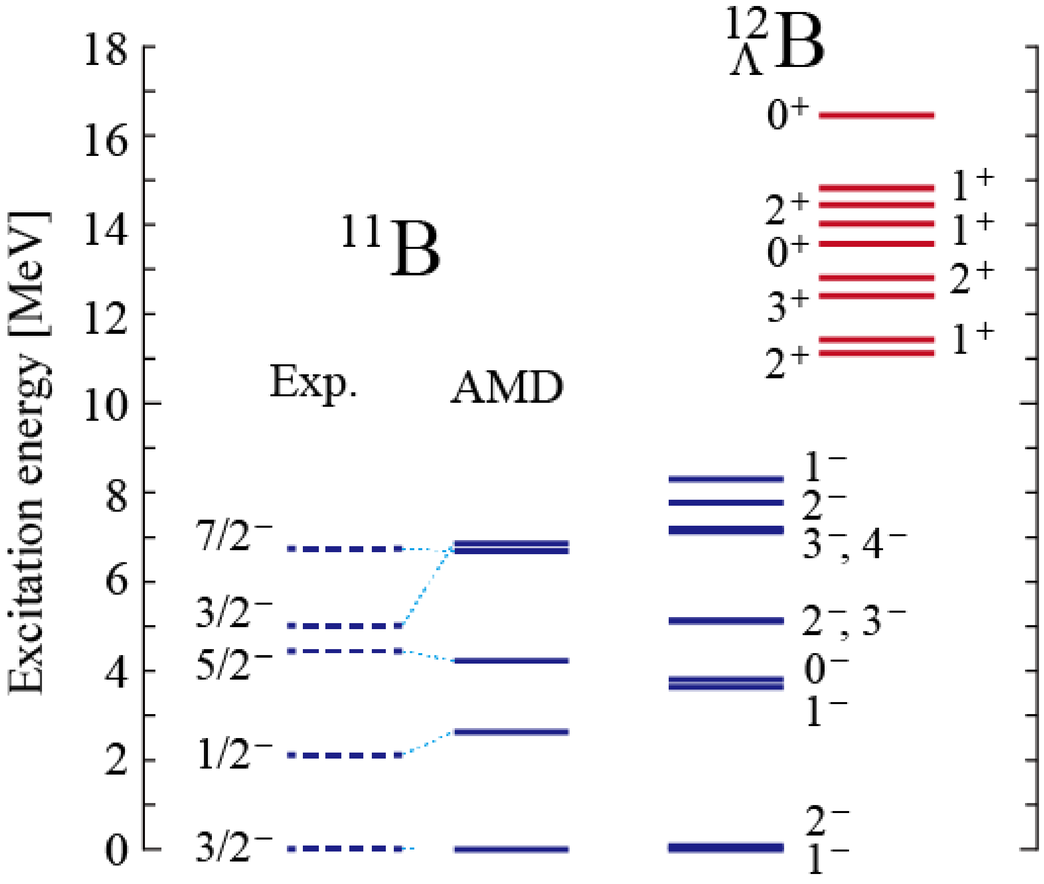

# Comparison with shell model + DWIA calc

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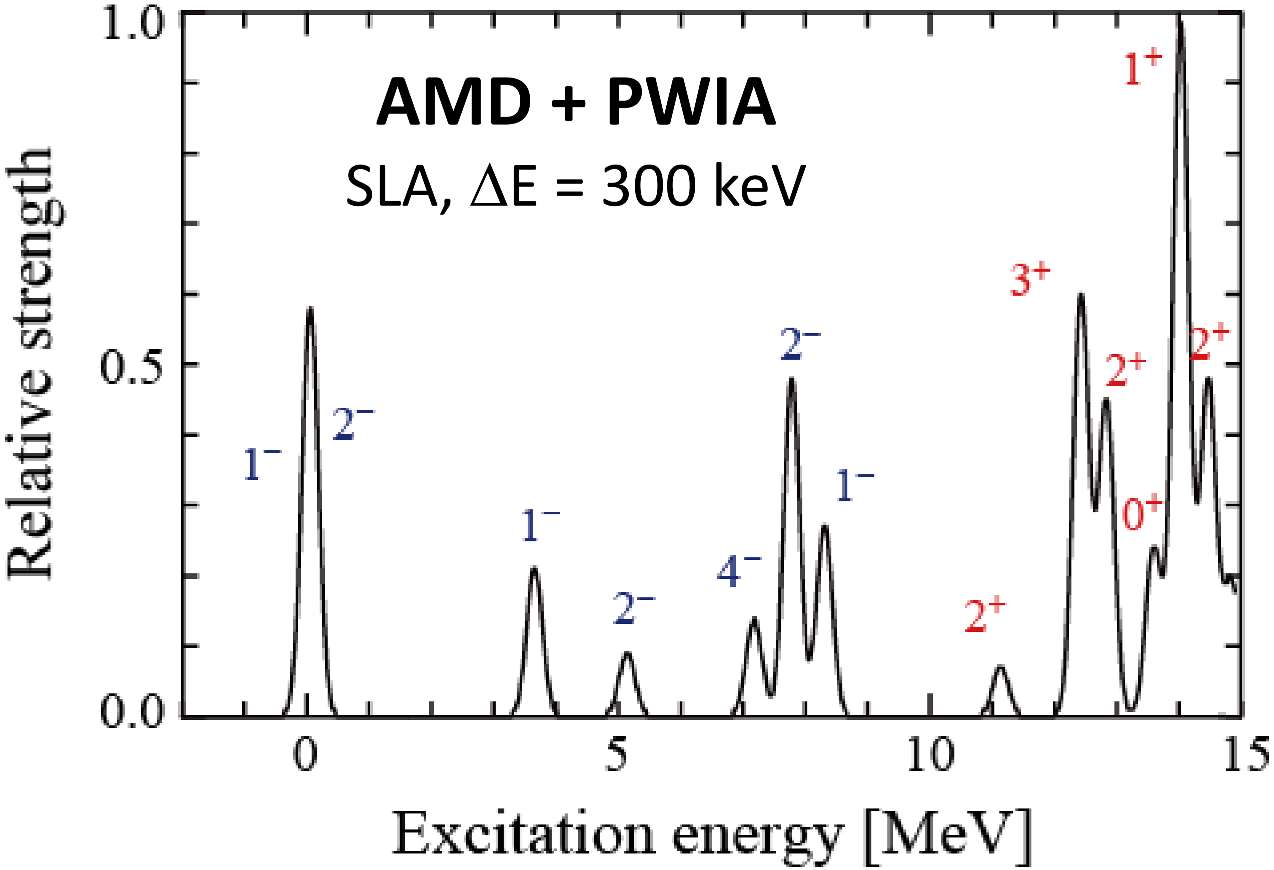
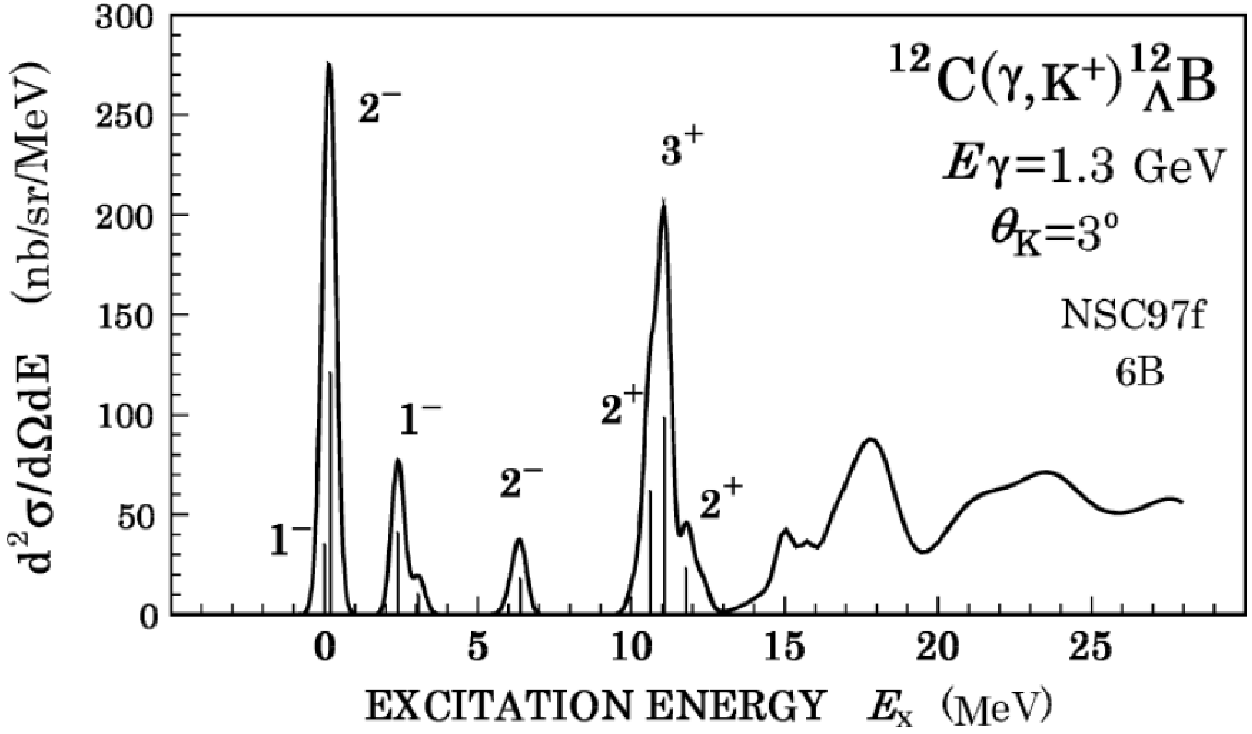



**Shell model + DWIA, SLA**



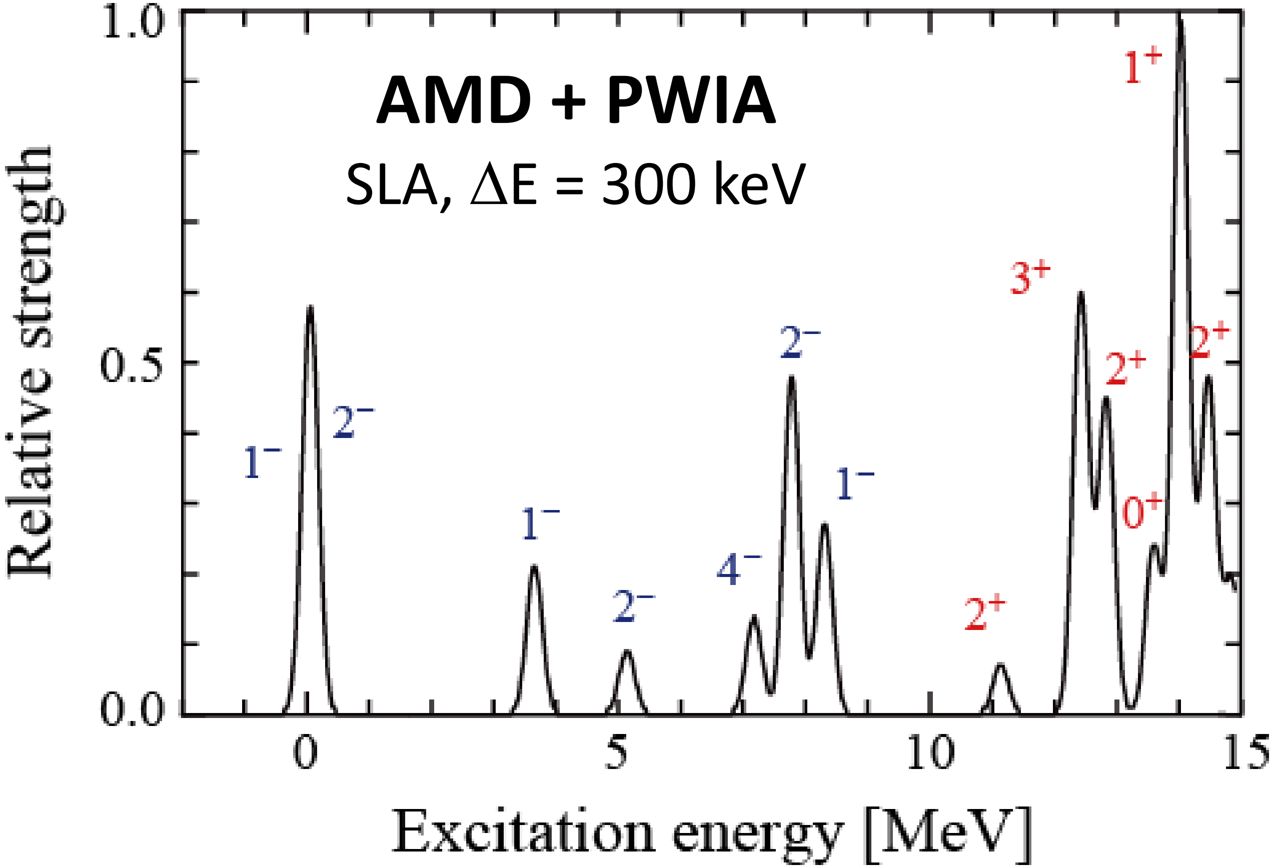

T. Motoba *et al.*, PTP**185**, 224(2010)

Several large peaks appear,
but relatively small for the ground-state doublet

# Comparison with experiments

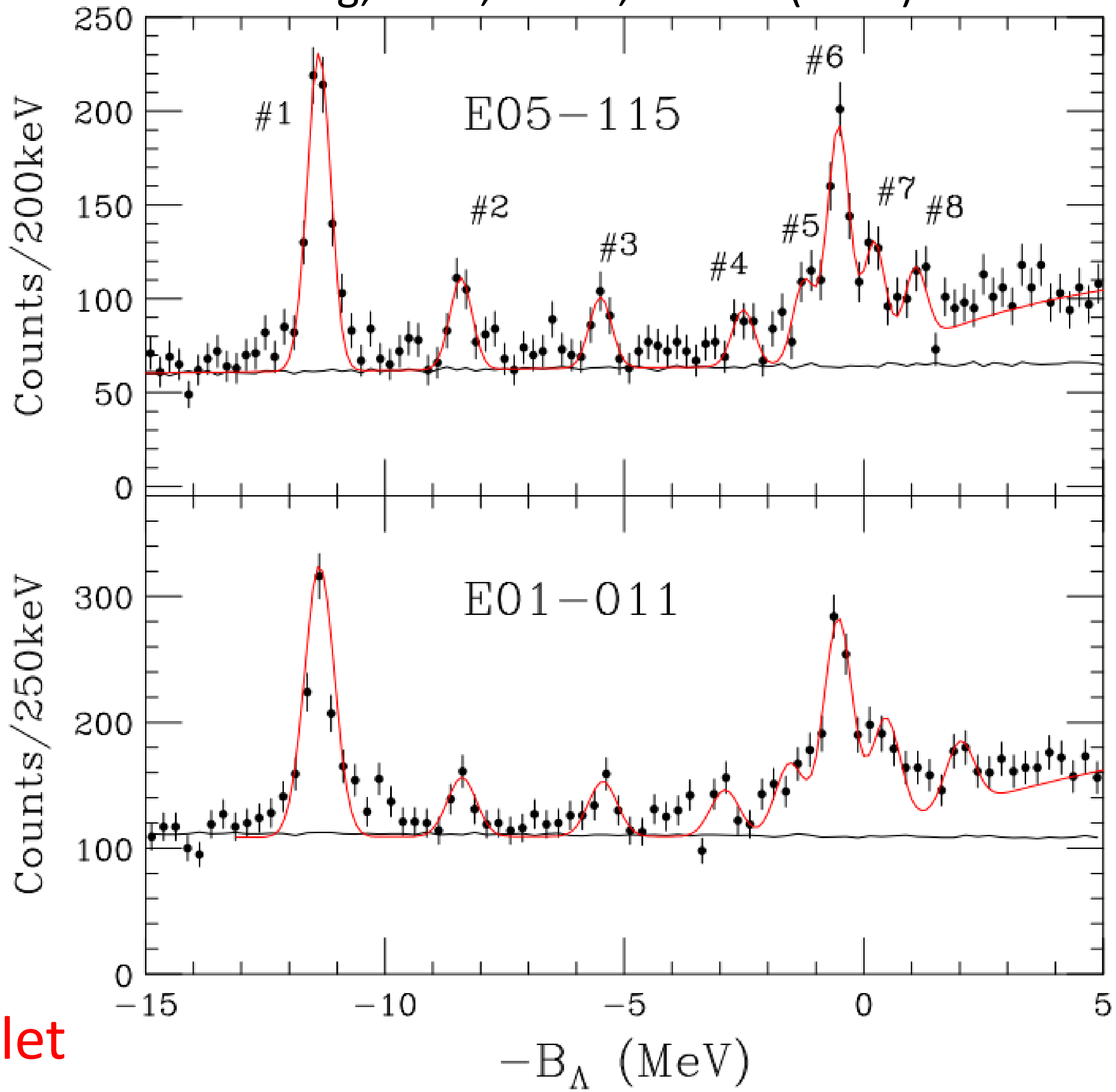
## $^{12}C(\gamma, K^+)^{12}{}_{\Lambda}B$

$E\gamma$ = 1.3 GeV, $\theta_K$ = 3 deg




Experiments at JLab

L. Tang, et al.,PRC**90**, 034320(2014)

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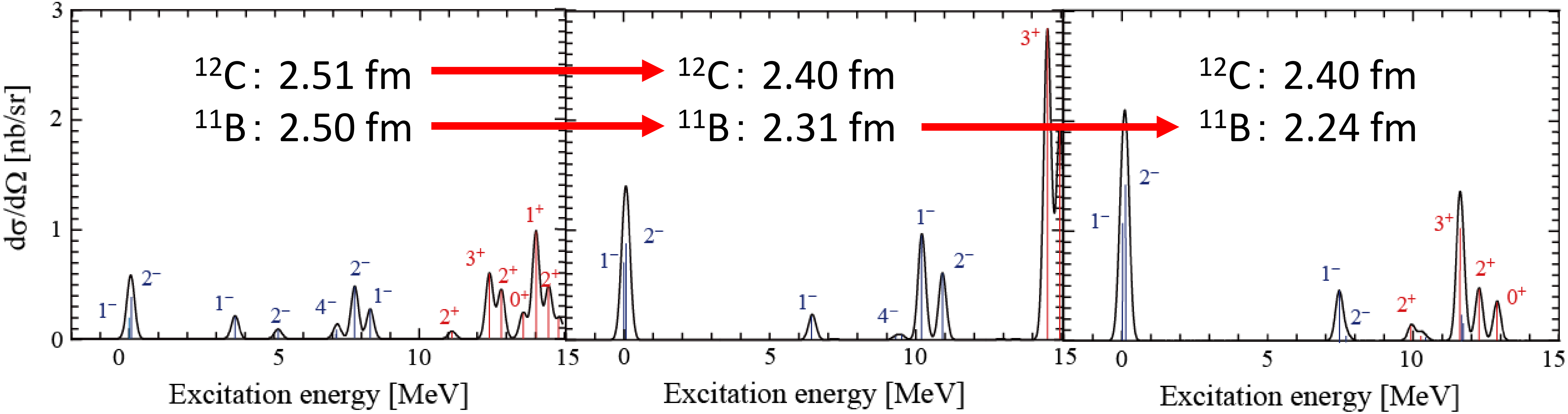
but relatively small for the ground-state doublet

# Results: Dependence of nuclear radius

- Nuclear radii of target ($^{12}C$) and core ($^{11}B$) nuclei are overestimated
- Tunning of nuclear radii by changing width parameter of Gaussian packets

$^{12}C(\gamma, K+)^{12}{}_{\Lambda}B$, HyperAMD + PWIA

**Without tuning** **Slightly tuned in $^{12}C$ & $^{11}B$** **Further tuned for $^{11}B$**



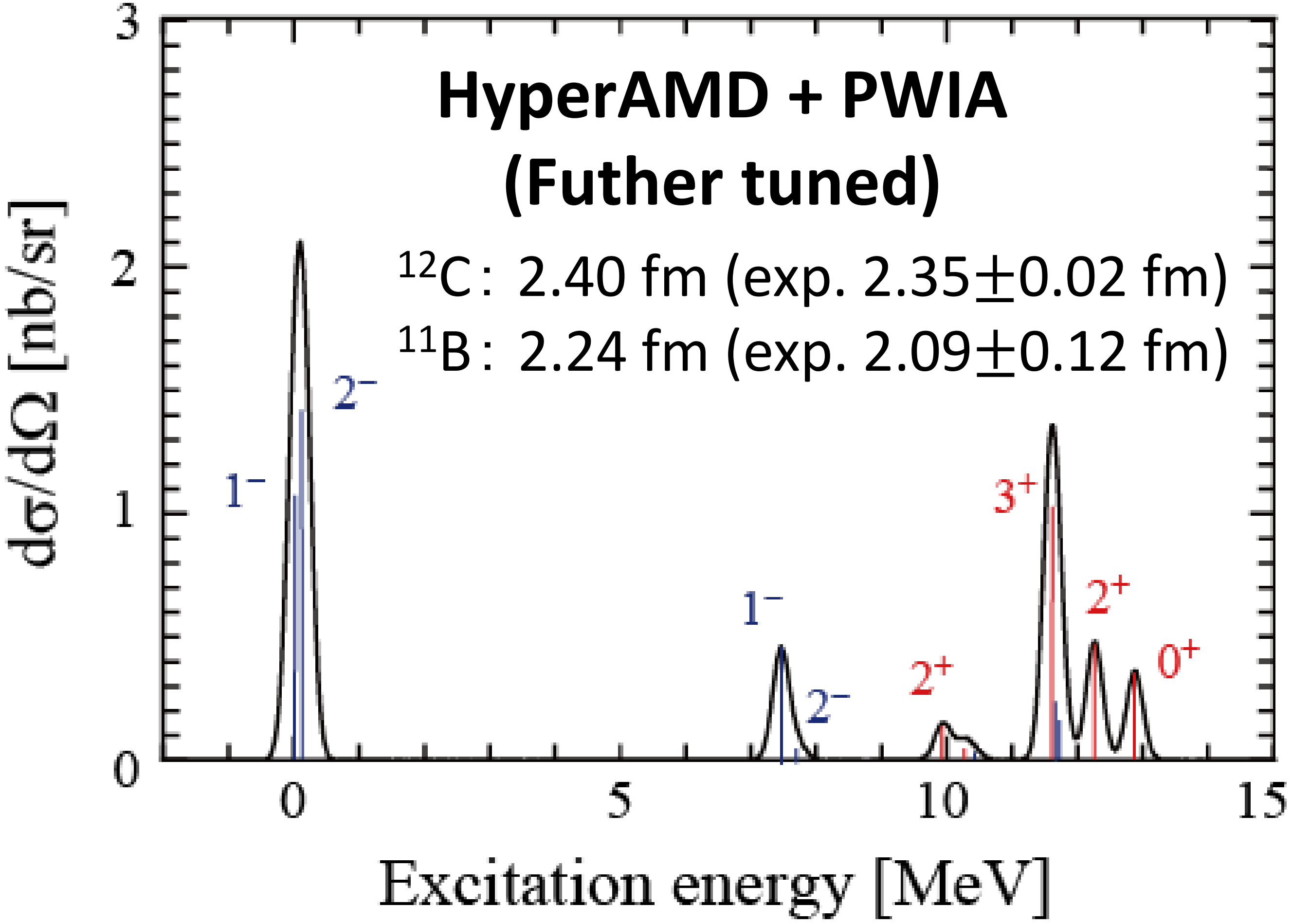

$^{12}C$: exp. 2.35±0.02 fm

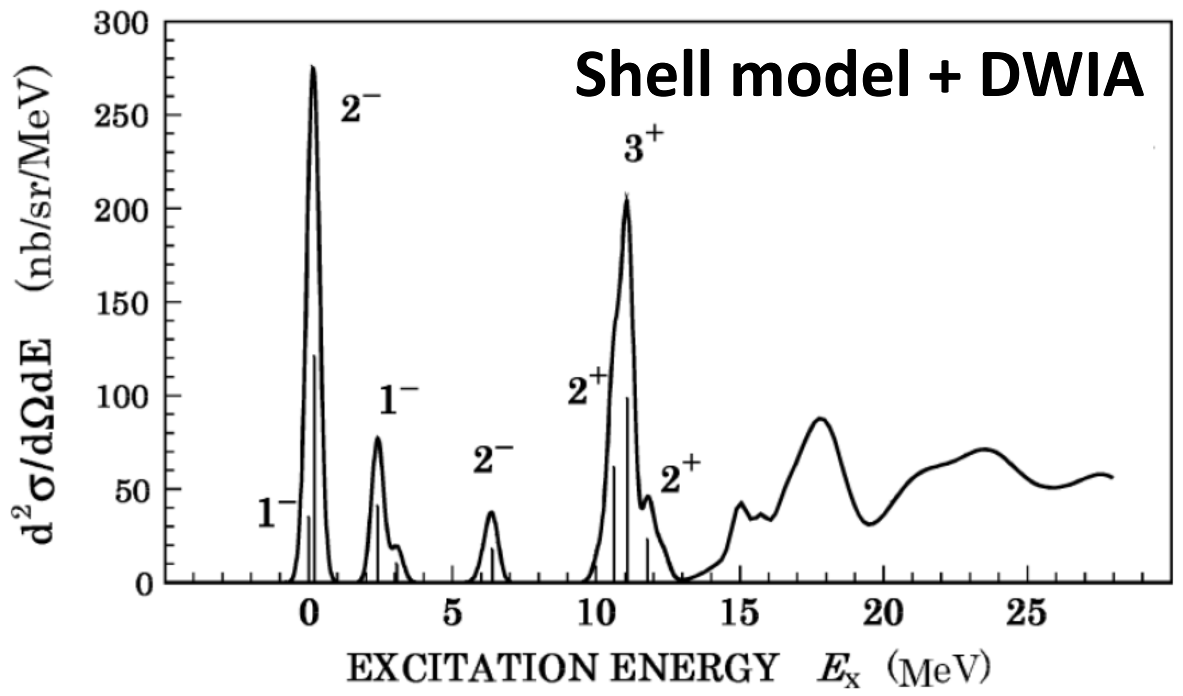
$^{11}B$: exp. 2.09±0.12 fm

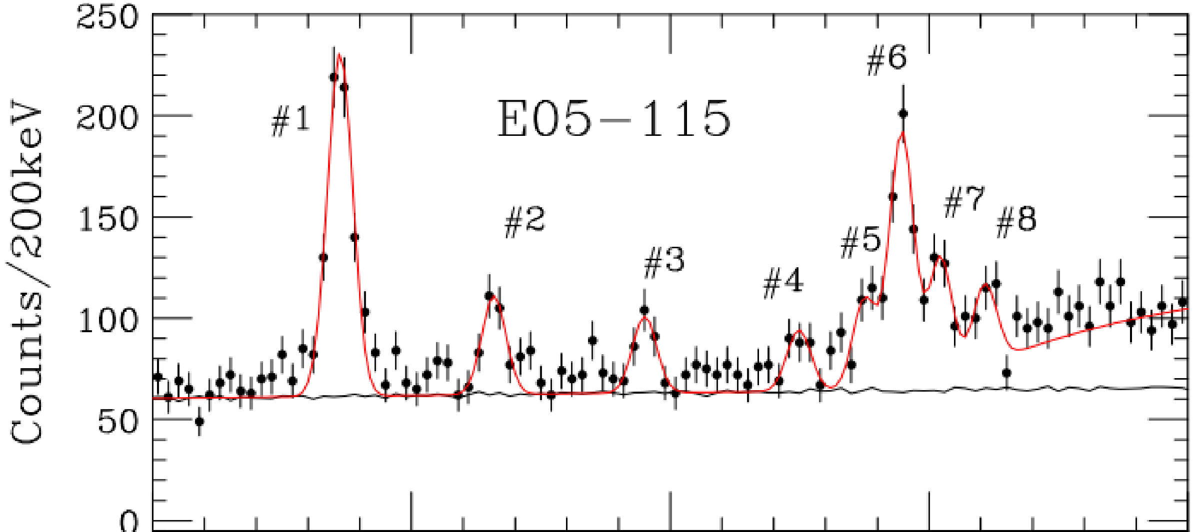
# Results: Further tuning

**$^{12}C(\gamma, K^+)^{12}{}_{\Lambda}B$**

$E\gamma$ = 1.3 GeV, $\theta_K$ = 3 deg



# Summary

- Coupling of $\Lambda$ particle to core nuclei shows unique structure

- To extract structure info from production cross section of hypernuclei, we perform reaction calculation based on HyperAMD wave functions

- HyperAMD + PWIA as the 1st step:
  - Comparison with observed data & shell model+DWIA calc for $^{12}C(\gamma, K^+)^{12}{}_{\Lambda}B$

    → To reproduce radii of target and core nuclei is important

- Future plans
  - Detailed analysis for $^{12}C(\gamma, K^+)^{12}{}_{\Lambda}B$
  - Extension to DWIA, application to other reactions